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Laminar flow in circular tube with internal solidification of a binary mixture

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ABSTRACT

Laminar flow heat transfer in circular tube with internal solidification of a binary mixture of paraffin is investigated numerically. The problem of the solidification process is formulated using the enthalpy–porosity based method. The influence of various parameters such as: wall temperature, inlet maximal velocity, initial temperature and initial concentration of the binary mixture, on the solidification process is analyzed in detail. It is found that the initial temperature has less important effect on the solidification progress than the wall pipe temperature, the initial concentration and the inlet velocity of the binary mixture.

1. Introduction

The solidification of liquids flowing through pipes under laminar and turbulent flow conditions is encountered in many engineering applications such as refrigeration systems design, hydraulics, cryogenics, liquid metal technology, nuclear, mechanical and chemical engineering [1]. The earliest interesting investigation on this subject was reported by Zerkle and Sunderland [3]. Hirschberg [2] studied the steady-state flow of a liquid through a tube under freezing conditions by assuming a constant pressure drop and solid-phase shell of uniform thickness. Zerkle and Sunderland [3] removed these restrictions and were able to show that when entrance effects are neglected and a parabolic velocity profile assumed, the steady-state problem of freezing in a tube reduces to the classical Graetz problem [4,5]. Hwang and Sheu [6] extended their work and considered the problem of liquid solidification in the combined hydrodynamic and thermal entrance region of laminar flow in a circular tube. They found that the theoretical heat transfer results of Zerkle and Sunderland [3] are in good agreement with their combined entry length theory. Ozisik and Mulligan [7] presented a solution for transient freezing of laminar flows inside channels, assuming a constant wall temperature. They applied integral transform to obtain the transient development of the ice-layer inside the tube. Bilenas and Jiji [8] reported solutions of transient freezing in tubes utilizing both numerical and approximate variational techniques. The combined thermal and hydrodynamic development during solidification in an isothermal tube has been analyzed by Hwang and Sheu [6]. Under the assumption that the axial variation in solid layer thickness was small, they were able to reduce the problem to the combined entry region problem without solidification. Chida [9] calculated numerically, under consideration of axial conduction, the steady state ice-layer thickness. Wei and Güçeri [10] used the vorticity-stream function approach and a Landou transformation to study the problem of transient freezing of laminar flows inside a circular pipe. A mathematical model was presented by Weigand and Beer [11] to predict the steady-state ice-layers inside a parallel plate channel or a circular pipe containing a turbulent flow. The effect of arbitrary entrance velocity distribution upon the shape of the ice layers was examined. A two dimensional axisymmetric numerical model has been developed by Jalali and Najafi [12] to analyze the solidification process of water in a cylindrical pipe. A wide range of the pipe wall temperature, mean velocity and temperature of fluid at the inlet of pipe has been taken into account to explain the effects of these parameters on the freezing of material.

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The study of axial growth of solid layer inside pipes is essential to predict the pressure built-up in the flow circuit. As a class, these problems are physically difficult to categorize because of the myriad of complex mechanisms which are oftentimes involved. One inherent difficulty in solving problems of this type is that the solid–liquid interface moves in time and is not known a priori [13,14]. Another difficulty arises in determining the liquid flow through a continuously deforming domain [15].

In situation involving the solidification of a pure substance, the system is characterized by a distinct solid–liquid interface whose position is neither fixed in time nor known a priori. The motion of the solidification front is determined by the rate at which heat can be transported through the solid. The situation may be further complicated by the existence of convection in the liquid.

The situation is radical different if the liquid is a multicomponent solution like a mixture of paraffins. In this case the solidification takes place over a range of temperatures, that range being a function of the local mixture composition [16,17]. Additionally, redistribution and consequent diffusion of species upon solidification arises. The result is the formation of separate zones-pure solid and pure liquid zones plus a mushy zone where solid and liquid coexist in thermal equilibrium. The solidification rate may now depend both on energy transport and species diffusion.

The *n*-alkanes and their mixture are excellent substances for the storage of energy [18,19]. The *n*-alkanes with 11–50 carbon atoms have melting points from −25 °C to +92 °C, which is in the range where many applications are found. In practice, for storage applications mixtures of alkanes are used rather than the pure substances themselves, especially if their "temperature windows" (temperature difference between liquidus and solidus) at a certain composition are narrow. First of all, mixtures offer the opportunity to tune in on the desired temperature level. For example, by combining *n*-pentadecane and *n*-hexadecane, mixtures with thermal windows less than 2 °C can be designed than range from about 10 °C to about 16 °C. Secondly, the use of mixtures is favored because they are less expensive than their pure components [20,21].

In order to successfully formulate the mathematical description of melting and solidification of the mixtures of alkanes, it is important to know exactly how these mixtures melt in relation to the pure components. Binary mixtures of *n*-alkanes serve as an important model system for understanding not only solidification, but also as a model for the investigation of the effect of heat transfer and initial concentration of the binary mixture on the solidification process.

To our knowledge, there exists no numerical treatment of the conservation equations for laminar flow with solidification of binary mixture of paraffin inside a tube. The aim of this paper is to propose a numerical model to describe the heat transfer of a binary mixture (hexadecane–tetradecane) during the solidification process for laminar flow in a circular duct with constant temperature. The problem of the solidification process is formulated using the enthalpy–porosity based method. Several simulations were conducted to investigate the effects of different parameters such as: wall temperature, inlet maximal velocity, initial temperature and initial concentration of the binary mixture.

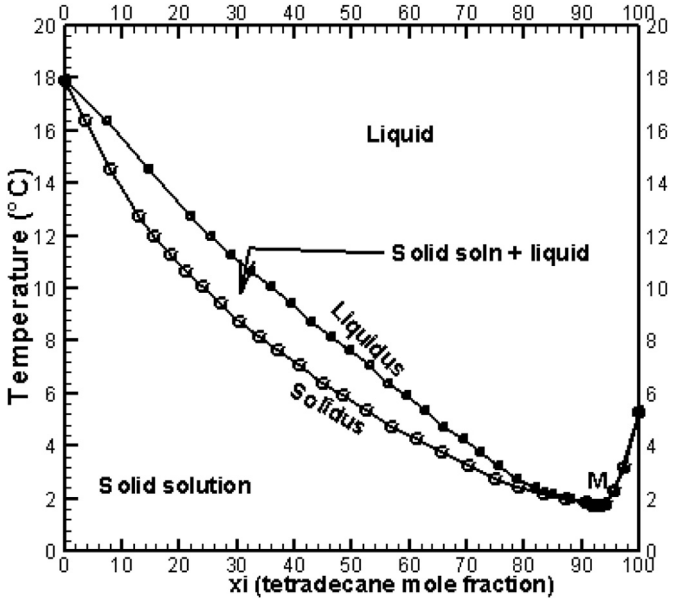

Fig. 1. The liquid–solid phase diagram of binary mixture system of $C_{14}H_{30}$ and $C_{16}H_{34}$ [19].

2. Liquid–solid phase diagram for binary mixture system of $C_{14}H_{30}$ and $C_{16}H_{34}$

Fig. 1 is the temperature-composition phase diagram for the binary system of tetradecane–hexadecane. The upper curve is the liquidus or freezing points curve as obtained by Ref. [21]. The lower curve is the solidus or melting points curve, obtained by the calculation UNIFAC method. Any system represented by a point above the liquidus is completely at the liquid state, and any point below the solidus is completely at the solid state. A point within the area enclosed by the liquidus and solidus curve indicates an equilibrium mixture of liquid and solid solution. These two curves approach and touch at point M. The azeotropic point of the hexadecane–tetradecane mixture occurs at 92% of tetradecane, and the phase change temperature at this point is approximately 1.8 °C.

3. Physical model and basic equations

The liquid flowing in laminar flow in a tube is assumed to have a uniform temperature T_i and a mean velocity U at the inlet. Initially the wall is maintained uniformly at T_i. In the region $x < 0$, the wall temperatures are kept constant at values above the liquidus temperature of the binary mixture and the velocity profile is assumed to be fully developed. At time $t = 0$, the wall in region $x \geq 0$ is suddenly lowered to a constant temperature T_w which is lower than the solidus temperature $T_{solidus}$ of the binary mixture and maintained at that temperature for $t > 0$. As a result, surface solidification starts in the region $x \geq 0$ and the thickness of the solidified shell is caused to increase with both time and position along the tube. Fig. 2 illustrates the geometrical model of a straight tube of 3m length (L) and constant transversal area of diameter D of 0.05m. The following assumptions are made:

- The flow is assumed to be axisymmetrical and incompressible.
- The liquid is Newtonian and the viscous dissipation is neglected.

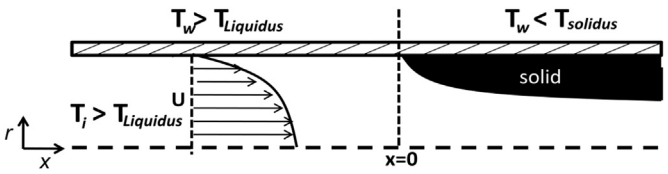

Fig. 2. Schematic model and coordinate system.