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Lattice model applied to the fracture of large strain composite

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Abstract

An improved lattice model is developed to simulate the fracture behavior of large strain composite. Based on equivalent relation between the continuum and the lattice model for small deformation, the equivalent relation between large strain continuum and improved lattice model is established by introducing large strain elastic law into the lattice system. The theory can simulate large deformation. The program of large strain lattice model simulates several representative problem of large strain elasticity. The results agree with the theoretical results. Assumed failure criterion is used to describe the fracture process of large strain elasticity and large strain composite. The improved lattice model provides an effective method for fracture simulation of large strain composite.

Keywords: Solid mechanics; Numerical method; Improved lattice model; Large strain composite; Fracture

1. Introduction

Much has been done on large strain elasticity [1–8]. Because of lattice aberrance and complexity of the fracture process, current numerical methods have limitations in simulating fracture of large strain composite. The lattice model does not need lattice integral, and can successfully simulate fracture behavior for small deformation [9–20] and some nonlinear problems [20,21]. Hence the lattice method can be applied to the large strain case and overcome limitations of current numerical methods. The present work attempts to deal with this problem. Large strain, geometry nonlinearity and material nonlinearity are included. Since the lattice model whose base elements are spring and do not need lattice integral, it is easy to simulate geometry for large deformation. However, it is more difficult to introduce material nonlinearity into the lattice system. This will considered in the work to follow by using the equivalent relation between the continuum and the improved lattice model for small deformation. This is accomplished by using an equal ratio method to introduce large strain elastic law into the lattice system.

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2. Stiffness of extended lattice model

2.1. Stiffness of poles in small deformation case

Two-dimensional problems are considered. The pole is selected as the base element of the lattice model which entails uniaxial stress only and use triangle elements to divide the lattices. This enhances homogeneity and stability. The determination of the stiffness of poles in the small deformation case will be given next.

Developed in [16] is an integral method by obtaining the element's stiffness tensor which is equivalent to the continuum stiffness tensor using the energy principle.

It is well known that the basic idea of setting up the lattice model is by using the equivalent of strain energy stored in a unit cell of a lattice of volume V

$$\boldsymbol{W}_{\text{cell}} = \boldsymbol{W}_{\text{continuum}}, \tag{1}$$

where the energy of the cell and its continuum equivalent are given by

$$\boldsymbol{W}_{\text{cell}} = \sum_b \boldsymbol{W}_b = \frac{1}{2}\sum_b^{|b|}(\underset{\sim}{\boldsymbol{F}} \cdot \underset{\sim}{\boldsymbol{u}})^{(b)}, \tag{2}$$

$$\boldsymbol{W}_{\text{continuum}} = \frac{1}{2}\int_V \underset{\sim}{\boldsymbol{\sigma}} \cdot \underset{\sim}{\boldsymbol{\varepsilon}}\, \mathrm{d}V, \tag{3}$$

In Eq. (2), b stands for the bth pole, and $|b|$ for the total number of poles. In the two-dimensional (2D) setting, the volume is an area of unit thickness. Limiting the discussion to linear elastic poles and spatially linear displacement field $\underset{\sim}{u}$ (i.e. uniform strain fields $\underset{\sim}{\varepsilon}$), Eqs. (2) and (3) become

$$\boldsymbol{W}_{\text{cell}} = \frac{1}{2}\sum_b^{|b|}(k\,\underset{\sim}{\boldsymbol{u}} \times \underset{\sim}{\boldsymbol{u}})^{(b)}, \tag{4}$$

$$\boldsymbol{W}_{\text{continuum}} = \frac{1}{2}\underset{\sim}{\bar{\boldsymbol{\varepsilon}}} \times \underline{\boldsymbol{C}} \times \underset{\sim}{\bar{\boldsymbol{\varepsilon}}}. \tag{5}$$

In Eq. (4) $\underset{\sim}{\boldsymbol{u}}$ is a generalized pole displacement and k is the corresponding spring constant. The next step, which will depend on the particular topology of the unit element and on the particular model of interactions, will involve making a connection between $\underset{\sim}{\boldsymbol{u}}$ and $\underset{\sim}{\varepsilon}$ such that C can be derived from Eq. (1). The corresponding procedure and resulting formulas are given below for the elastic problem using the triangular network.

In the continuum setting the constitutive law is

$$\boldsymbol{\sigma}_{ij} = \boldsymbol{C}_{ijkm}\boldsymbol{\varepsilon}_{km}, \quad i,j,k,m = 1,2 \tag{6}$$

while the balance equation is

$$\boldsymbol{\sigma}_{ij,j} = 0, \tag{7}$$

The Navier's equation for the displacement $\boldsymbol{u}_i$, is,

$$\mu'\boldsymbol{u}_{i,jj} + \kappa\boldsymbol{u}_{j,ji} = 0, \tag{8}$$

In Eq. (8) μ' is defined by $\boldsymbol{\sigma}_{12} = \mu\boldsymbol{\varepsilon}_{12}$, which makes it the same as the classical three-dimensional shear modulus. On the other hand, $\kappa$is the (planar) two-dimensional bulk modulus, which is defined by $\boldsymbol{\sigma}_{ii} = k\boldsymbol{\varepsilon}_{ii}$.

Approximated locally homogeneous media will be used. Consider a regular triangular network in Fig. 1 with central force interactions only. Hence, or each bond b, there results

$$\boldsymbol{F}_i = \boldsymbol{\Phi}_{ij}^{(b)}\boldsymbol{u}_j, \tag{9}$$

where

$$\boldsymbol{\Phi}_{ij}^{(b)} = \alpha^{(b)}\boldsymbol{n}_i^{(b)}\boldsymbol{n}_j^{(b)} \tag{10}$$