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Meshless simulation of crack propagation in multiphase materials

Y.P. Chen [a], A. Eskandarian [a], M. Oskard [b], J.D. Lee [a,*]

[a] *School of Engineering and Applied Science, The George Washington University, Washington, DC 20052, USA*
[b] *Turner-Fairbank Highway Research Center, Federal Highway Administration, US Department of Transportation, McLean, VA 22101, USA*

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Abstract

The material body considered in this work consists of multiphases. Digital imaging data are taken as the input to specify the configuration and composition of the specimen. Meshless method is demonstrated as a superior numerical tool to analyze crack initiation and propagation in multiphase material. A fracture criterion, based on the ratio of the opening stress over the material toughness distributed in front of the crack tip, is proposed to determine the direction of crack propagation of mixed mode fracture problem in multiphase material. Numerical results are presented and discussed.

Keywords: Multiphase; Meshless method; Crack propagation; Fracture criterion

1. Multiphase materials

Industry materials usually consist of multiphases, such as alloy, ceramics, composite material, polycrystalline solid, and concrete, etc. The boundaries between phases are usually irregular and random [1]. Some of materials usually consist of defects such as micro-crack, voids, and dislocations. Therefore, to say the least, it is very difficult to model and to perform numerical simulation of the detailed feature of multiphase material. Computational material models (constitutive relations, failure criteria) are generally obtained through macroscopic experiments statistically, and are only valid under certain similar conditions.

On the other hand, digital imaging data from CT, ultrasound, MRI, etc. are abounding. CT, also known as CAT scanning or X-ray computed topography, is a completely nondestructive technique that enables one to visualize detailed features in the interior of opaque solid objects and to obtain information on their 3-D geometry and composition. In CT, cross sectional images are generated by projecting a thin beam of X-ray through one plane of an object from many different angles. A 2-D image of a section or a slice of a 3-D object usually has 512×512 pixels. The value of each pixel is a measure of the reduction in X-ray intensity and energy, which in turn is a measure of the density of the material at that point. Therefore, the values at the pixels

* Corresponding author.
E-mail address: jdlee@gwu.edu (J.D. Lee).

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can be taken as the input to specify the configuration and composition of the specimen.

2. Meshless methods

Meshless methods construct approximations entirely in terms of nodes. A *weight function*, which plays an important role in the performance of the methods, is used in all varieties of meshless methods. The *compact support of weight functions*, also called the domain of influence of a node, gives a local character to the meshless methods. The weight function is nonzero within the domain of support and zero outside of the domain of support. The commonly used supports are discs and rectangles, as shown in Fig. 1.

The character that meshless methods can be constructed solely in terms of nodes without the need of a mesh distinguishes meshless method from finite element (FE) method, which requires a highly structured mesh. For a variety of problems with large deformation, moving discontinuities, or in optimization problems where re-meshing may be required, meshless methods are very attractive [2–4]. The meshless methods employed in this paper is based on the moving least square technique in which the approximation of any scalar-valued function, $\widetilde{U}(\mathbf{x})$, can be expressed as an inner product between a vector of shape functions, $\boldsymbol{\Phi}(\mathbf{x})$, and a vector of nodal values, $\mathbf{U}$, as

$$\widetilde{U}(\mathbf{x}) = \boldsymbol{\Phi}(\mathbf{x}) \cdot \mathbf{U}, \tag{1}$$

which has the same form as in the FE method. However, there is a fundamental difference between FE method and meshless method. In meshless method, Eq. (1) is an approximation rather than an interpolation, i.e., $\widetilde{U}(\mathbf{x}_i) \neq U_i$. This character requires special and careful treatments of essential boundary conditions, mirror symmetries, and moving discontinuities, such as crack propagation [4,5].

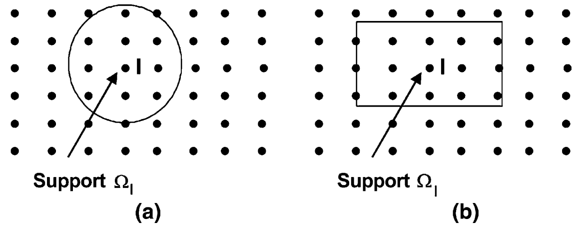

Fig. 1. The commonly used supports of node I: (a) circular and (b) rectangular.

In meshless method material properties are specified at each node, i.e.,

$$\boldsymbol{A}_\alpha = \boldsymbol{A}(\boldsymbol{X}_\alpha). \tag{2}$$

Then the value and the slopes of any variable at a generic point $\boldsymbol{X}$ can be expressed in terms of its neighboring nodal values as

$$\boldsymbol{A}(\boldsymbol{X}) = \sum_\alpha \boldsymbol{\Phi}_\alpha(\boldsymbol{X})\boldsymbol{A}_\alpha, \quad \nabla\boldsymbol{A} = \sum_\alpha (\nabla\boldsymbol{\Phi}_\alpha)\boldsymbol{A}_\alpha. \tag{3}$$

3. Crack propagation

In two-dimensional fracture problems, Mode I fracture may lead to self-similar crack extension due to symmetry. In general case, especially in case of multiphase material, we encounter mixed mode fracture problems. Therefore, to determine the direction of crack extension is an unavoidable task. Usually, we use the maximum opening stress criterion or the maximum energy release rate criterion to determine the direction of crack propagation. For example, using maximum opening stress criterion, the current crack tip will extend to $\{r_c, \theta\}$ if the opening stress $t_{\theta\theta}$ is maximum at $\{r_c, \theta\}$, where $r_c > 0$ is small but finite constant. One may consider that $t_{\theta\theta}(r_c, \theta)$ is the driving force for crack extension along the arc, as a function of θ, with a radius r_c with respect to the current crack tip. If the material is homogeneous, the maximum opening stress criterion is reasonable, i.e., the information of the driving force is enough to determine the direction of crack extension. However, if the material is inhomogeneous, one has to consider the resistance, i.e., the toughness, distributed in front of crack tip.

In this work, it is proposed that the current crack tip will extend to $\{r_c, \theta\}$ if the ratio

$$R(r_c, \theta) \equiv \frac{t_{\theta\theta}(r_c, \theta)}{t_c(r_c, \theta)}, \tag{4}$$

reaches a maximum at $\{r_c, \theta\}$, where t_c is the toughness associated with the opening stress (cf. Fig. 2). Crack propagation process can be viewed as a changing of crack tip with a moving barrier following the advancing of the crack tip. It is noticed that both $t_{\theta\theta}(r, \theta)$ and $t_c(r, \theta)$ are continuous functions of r and θ in meshless method; However, in finite element method, at the element boundaries displacements are continuous but strains, stresses, material properties are discontinuous. This is another advantage of meshless method, especially when invokes