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Triaxial compressive behavior of rock with mesoscopic heterogenous behavior: Strain energy density factor approach

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Abstract

The strain energy density factor approach is used in conjunction with a micromechanics model to investigate the condition and direction of shear failure for brittle rock subjected to triaxial compression. Moderate confinement in addition to localized deformation and damage are considered. Quantified are the effects of the various geometric and load parameters that involve the interaction of microcrack, friction and the confining pressure such that the path of the wing crack is taken into account. The influence of all microcracks with different orientations are introduced into the constitutive relation. The closed-form solution for the complete stress–strain relation of rock containing microcracks is obtained. It is shown that the complete stress–strain relationship includes linear, nonlinear hardening, rapid stress drop and strain softening effects. The theoretical results show that deviation of the direction of wing cracks from the line of the pre-existing crack decreases with increasing confinement pressure and friction coefficient. Theoretical predictions and experimental results show good agreement.

Keywords: Triaxial compression; Moderate confinement; Strain energy density factor approach; Mesoscopic heterogenous of brittle rock; Deformation and damage localization; Stress–strain relation; Shear failure

1. Introduction

Fully and partially underground rocks are often under complex multi-axial stress state and they are susceptible to yield and failure. This study is concerned with the mechanical behavior of the materials' response to load. Identified are the failure conditions in different stress states. Design criteria are obtained for the development of these construction materials. This involves the determination of the constitutive relation and the prevailing stresses under compression in addition to the constraint condition. Damage of the rock structure is analyzed to determine the necessary design requirement for the rock support. As a rule rock material is inhomogeneous containing initial defects that include grain boundaries, microcracks and pores. Experimental observations show that axial splitting due to the propagation of a few dominant cracks occurs under uniaxial compressive loads or biaxial compressive loads with low lateral compressive stress. Shear failure occur by the

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formation of a single fault or multiple faults of cracks that happens in the presence of a moderate confinement [1–4]. These observations have led researchers to focus attention on analyzing the behavior of axial splitting and shear failure by experiments and analyse [3–20]. Investigations of the mechanical mechanism associated with shear failure are still very limited. Several micromechanics crack models such as the cylindrical pore model [14], dislocation pile-up model [15] and frictional sliding crack model [5–9] have been proposed. Among them, the frictional sliding crack model has been widely applied to study the behavior of cracks in an effort to study the mechanical properties of materials. To the author's knowledge, no micromechanical model has been presented to calculate the stress–strain relation due to shear failure with microcrack interaction for brittle rock with attention given to the localization of damage and deformation under triaxial compression. This paper aims to develop a micromechanical model for determining the effects of microcrack interactions related to shear failure and the associated material and loading.

The strain energy density factor approach and the frictional sliding crack criteria will be used. In particular, the condition and direction of localized damage and deformation will be examined. As an illustration, the theoretical stress–strain curves of Inada granite rock material subjected to triaxial compressive loads with moderate confinement will be found and compared the experimental results [21,22].

2. Theoretical model

The total strain increment may be splitted into elastic strain increment part $d\varepsilon_{ij}^{0}$, which is the strain increment in the rock material if there are no microflaws. The inelastic strain increment part $d\varepsilon_{ij}^{m}$ accounts for the inelastic deformation of the pre-existing microcracks. The sum is

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{0} + d\varepsilon_{ij}^{m} \tag{1}$$

The elastic strain increment is defined by

$$d\varepsilon_{ij}^{0} = S_{ijkl}^{0}\, d\sigma_{kl} \tag{2}$$

where S_{ijkl}^{0} is the elastic compliance tensor of the matrix material. The inelastic strain increment will be formulated as in [10]. This requires

$$d\varepsilon_{ij}^{m} = \frac{1}{V_0}\sum \frac{\partial f_a(\sigma, H)}{\partial \sigma_{ij}}\, d\xi_a \tag{3}$$

In Eq. (3), $f_\alpha(\sigma, \mathrm{H})$ is the set of thermodynamic forces conjugated to the internal variable ξ_α where σ_{ij} are the components of the stress tensor. H represents symbolically the current collection of ξ_α, V_0 denotes the volume of a representative volume element (RVE). The summation in Eq. (3) extends over all sites of the RVE where the microstructural rearrangement take place.

Consider a representative volume element featuring a mesoscopic length scale which is much larger than the characteristic length scale of microcrack but smaller than the characteristic length scale of a macroscopic specimen. Establish the global coordinate system (o–x_1x_2) and the corresponding local coordinate system (o–$x_1'x_2'$), in which x_1'-axis is parallel to the normal vector **n,** as shown in Fig. 1. It is assumed that the length of the pre-existing crack pp_1 is $2c$, its normal forms an angle θ with respect to the maximum compression direction, as depicted in Fig. 1. Assume there are N pre-existing microcracks, having a common average length $2c$. In order to investigate the condition and direction of shear failure for brittle rock subjected to triaxial compressive loads with moderate confinement, interaction of parallel cracks must be taken into account. Consider parallel cracks of identical length, periodically distributed in an infinite solid and the line that passes through their centers forms an angle α with respect to the maximum compression direction, as shown in Fig. 2.

2.1. The frictional sliding under triaxial compressive loads

For a microcrack, the normal and shear stresses actually acting on pp_1 is denoted by

$$\begin{aligned} \sigma_{11}^{r} &= \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta \\ \tau_{12}^{r} &= -\frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta \end{aligned} \tag{4}$$