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Nonhomogeneous Poisson process with nonparametric frailty

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ABSTRACT

The failure processes of heterogeneous repairable systems are often modeled by non-homogeneous Poisson processes. The common way to describe an unobserved heterogeneity between systems is to multiply the basic rate of occurrence of failures by a random variable (a so-called frailty) having a specified parametric distribution. Since the frailty is unobservable, the choice of its distribution is a problematic part of using these models, as are often the numerical computations needed in the estimation of these models. The main purpose of this paper is to develop a method for estimation of the parameters of a nonhomogeneous Poisson process with unobserved heterogeneity which does not require parametric assumptions about the heterogeneity and which avoids the frequently encountered numerical problems associated with the standard models for unobserved heterogeneity. The introduced method is illustrated on an example involving the power law process, and is compared to the standard gamma frailty model and to the classical model without unobserved heterogeneity. The derived results are confirmed in a simulation study which also reveals several not commonly known properties of the gamma frailty model and the classical model, and on a real life example.

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1. Introduction

A repairable system is a system which is repaired and put back in operation after failures. Traditionally, the literature on repairable systems is concerned with modeling of failure times, and typical data consist of an observed point process. The most commonly used models for the failure process of a repairable system are renewal processes (RP) and nonhomogeneous Poisson processes (NHPP). An RP, which is characterized by a renewal distribution describing the time between failures, and which is also called a perfect repair model, assumes that the system is as good as new after a repair and corresponds to assumption of replacement of a system. On the other hand, an NHPP, which is characterized by the rate of occurrence of failures (ROCOF), corresponds to the minimal repair assumption [3], for which it is assumed that the system after repair is only as good as it was immediately before the failure.

For complex systems only a small fraction is usually repaired at failures. In this case, the system is only as good as it was immediately prior to the failure and an NHPP is a more appropriate model than a renewal process. Moreover, the NHPP allows the modeling of trends of the failure interoccurrence times, for example, whether a system is improving or deteriorating. Therefore, this paper will concentrate on the modeling of repairable systems using nonhomogeneous Poisson processes.

Nonhomogeneous Poisson processes are widely used models in reliability. NHPPs are used, for example, in modeling of failures of gas turbine engines on offshore platforms in a recent paper [22] as well as for analysis of failures of hard drives in [26], failures of ozone analyzers in [10] or failures of NC machine tools in [25]. NHPPs can also be used for modeling and optimization of maintenance (for example, see, [18] or [11]) or for modeling software reliability [8] and they are also often used as a basic model which is adapted to a specific real world applications [24].

In many real life applications there is a substantial heterogeneity between apparently identical repairable systems, which cannot be described by observed covariates. This unobservable heterogeneity, which is often called frailty in the survival analysis literature, is usually modeled by multiplication of the basic ROCOF by a positive random variable taking independent values across the systems with unit mean and therefore described by its variance. In the traditional models with unobserved heterogeneity, it is necessary to define the distribution of the unobserved effects [1,15]. Since the modeled heterogeneity is unobservable, the choice of distribution of the unobserved effects is a problematic part of using these models, similar to the choice of prior distribution in Bayes models. Moreover, in fitting the traditional frailty model it is necessary to integrate out the unobserved effects from the likelihood which has to be done numerically in many

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cases. This can cause problems, especially in complicated models and large data sets. Suitable choice of the distribution of unobserved effects can give interesting general results, but generally the main quantity of interest is the variance of the unobserved effects. A large variance may either indicate deficiencies in choice of model, or that some important factors or covariates have not been taken into account, which may influence the model fit.

Another result from incorporating unobserved effects, which is often overlooked but which is important for predictions and which can provide important additional information about the modeled system, is the ability to estimate individual frailties. Unobserved effects can be viewed as unobserved covariates and comparison of estimates of the individual frailties can give indication about important covariates influencing the underlying modeled process.

Parametric frailty models are common in the literature and the so called gamma frailty model, in which the unobserved effects are assumed to be gamma distributed, is probably the most popular choice. Standard references for Poisson processes with unobserved heterogeneity, with particular emphasis on applications in reliability, are [12,15] and the monograph [23]. The recent reliability literature contains several studies which involve unobserved heterogeneity. General discussions involving frailty modeling can be found in [20,5,7] or [6], while [14] consider a concrete case including both observed and unobserved heterogeneity. The gamma frailty model is used in a study of degradation of locomotive wheel sets in [17], and in a study of crash frequency in [2]. The above references are all concerned with parametrically modeled heterogeneity. Treatments of nonparametric frailty models are mostly found in the biostatistics literature, but are still rather scarce (see, e.g., [21] and references therein). A different extension of NHPPs has recently been considered in [16], assuming that the ROCOF of a system at any time depends on the previously experienced number of failures. The relation between this kind of modified NHPP and the heterogeneous NHPP is explored in [4]. In fact these models are mathematically indistinguishable [1,4].

The main purpose of this paper is to develop a method for estimation of the parameters of a non-homogeneous Poisson process with unobserved heterogeneity with the aim to keep most of the advantages of the traditional models and at the same time eliminate the disadvantages of these models. More precisely, we aim at a method for analyzing unobserved heterogeneity which does not require parametric assumptions about the unobserved heterogeneity and which avoids the frequently encountered numerical problems associated with the standard models with unobserved heterogeneity. It should be noted that the present paper is significantly different from [5], which is discussing some of the consequences of overlooking unobserved heterogeneity using well-known gamma frailty model and power law process.

The main idea of the presented approach is to treat the individual frailties as unknown parameters and to estimate them directly from the given data, similarly as in fixed effects models (in ordinary regression), while keeping in mind that they are realizations of a random variable without making any restrictive assumptions about their distribution. In addition, properties of the nonhomogeneous Poisson processes are used to obtain other interesting results, mainly for estimation of the variance of the unobserved effects.

The present paper is organized as follows. Section 2 is the main section where the model is introduced in case of the power law process. Estimators of the parameters are derived together with estimators of their variances as well as estimators of the variance of the unobserved heterogeneity. The obtained results are compared to the standard approaches, either using the gamma frailty model, or assuming no heterogeneity. Section 3 describes generalization of the introduced method to other parametric intensities than the power law. Section 4 presents results of a comprehensive simulation study with the aim of empirical verification of the theoretical results of this paper. The detailed results of this study can be found in the supplementary material to this paper. Section 5 illustrates the functionality of the introduced method on a real life example. Some concluding remarks are given in Section 6. Appendix A is devoted to a deeper study of the estimator of the trend parameter in the power law process, which may be of independent interest, while Appendix B refers to supplementary material for this article.

2. The power law process with unobserved heterogeneity

The nonparametric frailty model will be introduced in the case of a power law process and the results will be compared to the gamma frailty model and to the classical model without any unobserved heterogeneity.

Let us consider m independent nonhomogeneous Poisson processes of power law type with unobserved heterogeneity, i.e., such that the rate of occurrence of failures (ROCOF) of the jth system ($j=1,\dots,m$) is given by

$$\lambda_j(t\,|\,z_j) = z_j a b t^{b-1}, \tag{1}$$

while the cumulative ROCOF is

$$\Lambda_j(t\,|\,z_j) = z_j a t^b. \tag{2}$$

Here $a>0$ and $b>0$ are the standard parameters of the power law process, while the z_j represent the unobserved heterogeneity for the jth system. It is assumed that the z_j's are independent and identically distributed realizations of an unobserved positive random variable Z with unit mean and finite variance. Note that the classical model (i.e., the model without unobserved heterogeneity) is included in this setting as a special case with $\mathrm{P}(Z=1)=1$.

Let the jth system be observed for a time τ_j, which is assumed to be a realization of a positive random variable τ independent of the random variable Z. Denote the number of events in this system by n_j, and let the event times in this system be denoted as t_{ij} ($j=1,\dots,m$, $i=1,\dots,n_j$), as illustrated in Fig. 1.

Then the likelihood of the jth system, conditional on the z_j, is given by [1]

$$L_j = \left(\prod_{i=1}^{n_j} \lambda_j(t_{ij}\,|\,z_j)\right) \exp\left(-\Lambda_j(\tau_j\,|\,z_j)\right) = \left(\prod_{i=1}^{n_j} z_j a b t_{ij}^{b-1}\right) \exp(-z_j a \tau_j^b). \tag{3}$$

2.1. The nonparametric frailty model

The nonparametric frailty model will not need any additional assumption about the distribution of the unobserved effects.

The total loglikelihood for the m systems based on (3) is given by (conditionally on z_j's, $j=1,\dots,m$)

$$l = \sum_{j=1}^{m}\left(n_j \log(z_j) + n_j \log(a) + n_j \log(b) + (b-1)\sum_{i=1}^{n_j}\log(t_{ij}) - z_j a \tau_j^b\right)$$

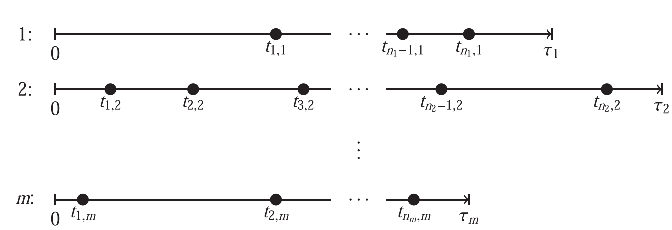

Fig. 1. Notation used in the mathematical model. Black dots denote failures.

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