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Age replacement policy based on imperfect repair with random probability

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ABSTRACT

In most of literatures of age replacement policy, failures before planned replacement age can be either minimally repaired or perfectly repaired based on the types of failures, cost for repairs and so on. In this paper, we propose age replacement policy based on imperfect repair with random probability. The proposed policy incorporates the case that such intermittent failure can be either minimally repaired or perfectly repaired with random probabilities. The mathematical formulas of the expected cost rate per unit time are derived for both the infinite-horizon case and the one-replacement-cycle case. For each case, we show that the optimal replacement age exists and is finite.

1. Introduction

As most of industrial systems become more complex and multiple-function oriented, most organizations expend a great amount of cost associated with equipment failure and its subsequent repair and replacement. Such an expenditure can typically reduced by the preventive maintenance (hereafter, PM) while the system is still in operation. Barlow and Hunter [2] and Barlow and Proschan [3] propose two types of PM policies. Under an age replacement policy, an operating system is replaced at age T or at failure, whichever occurs first while under a block replacement policy, the system is replaced at fixed time epochs of $T, 2T, 3T, \cdots$, and at failure.

The age replacement policy has been extensively studied by incorporating various types of repairs at failure and cost-structures for repair. Beichelt [4], Berg et al. [5], Block et al. [6] and Sheu et al. [18] consider the age-replacement problem with age-dependent minimal repair and different cost-structures. Cleroux et al. [9] and Bai and Yun [1] consider age replacement policy based both on the system age and the minimal repair cost. Sheu and Griffith [19], Sheu [16], Sheu and Chien [17], and Chien and Sheu [8] consider age replacement policy of system subject to shocks. Sheu [15], Sheu et al. [20], Juang and Sheu [11] and Sheu et al. [21] consider age replacement policy with age dependent replacement and random repair cost.

In most of literatures mentioned, it is noted that system is either minimally repaired or perfectly repaired at failures depending on repair cost or types of failures. That is notion of imperfect repair model proposed by Brown and Proschan [7]. Minimal repair restores the system to the state just prior to failure while perfect repair returns the system to the state as good as new one. That is, a failed system is either perfectly repaired or minimally repaired with probability p and $q(=1-p)$, respectively. Fontenot and Proschan [10] propose a modified age replacement model in which system is replaced by new one at age T and is either perfectly repaired with probability p or minimally repaired with probability $1-p$ at intermittent failures. Lim et al. [13] consider the case that the probability of perfect repair may not be predetermined in the most real situations and suggest Bayesian imperfect repair model in which the probability of perfect repair is considered as a random variable with a certain distribution.

In this paper, we consider age replacement policy with Bayesian imperfect repair. For our model, the system is replaced by new one at age T and when the system fails before age T, it is either perfectly repaired with random probability P or minimally repaired with random probability $1-P$. We formulate the expected cost per unit time for both the infinite-horizon case and the one-replacement-cycle case. We investigate the optimal age replacement policy which minimizes the expected cost rate per unit time.

The remainder of this paper is organized as follows. Section 2 describes the age replacement policy under consideration and investigates properties of hazard rate incurred by proposed age replacement policy. In Section 3, the expected cost per unit time

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Nomenclature

X	lifetime of a system
$F(t)$, $f(t)$, $r(t)$	Cdf, pdf and the failure rate of X, respectively
T	age for planned replacement
C_p, C_m, C_r	costs for perfect repair, minimal repair and replacement, respectively
P	r.v. representing the probability of perfect repair
$N(t)$, $L(t)$, $M(t)$	number of failures, number of perfect repairs and number of minimal repairs in $(0, t)$, respectively
Y_1, Z_1	waiting times at which the first perfect repair and the first minimal repair occurs, respectively
$H(t)$, $G(t)$	Cdfs of $Y_1 and Z_1$, respectively
$r_H(t), r_G(t)$	failure rates of $Y_1 and Z_1$, respectively
$B(T)$	expected long-run cost per unit time
Y_i^*	time duration between two successive renewals of system
R_i^*	total cost incurred over the renewal interval Y_i^*
$W(T)$	total cost per unit time between two successive replacement

for both the infinite-horizon case and the one-replacement-cycle case is formulated and the optimal replacement schedule is investigated. In Section 4, a numerical example is given to illustrate our results.

2. Age replacement policy

We consider the following age replacement policy of this system. A planned replacement with cost C_r is scheduled at the age T. At failure before T, the system is either perfectly repaired with probability P or minimally repaired with probability $1-P$. The probability of perfect repair is assumed to be randomly distributed with k possible values. That is, P has a discrete probability distribution which has the following probability mass function.

$P(P=p_i)=\pi_i$ for $i=1,2,\cdots,k$.

Costs for perfect repair and minimal repair are C_p and C_m, respectively. Life cycle of system is renewed at age T or at the first failure which is perfectly repaired, whichever comes first. After a replacement, the procedure is repeated.

When the prior distribution is one-point prior, the repair model and the age replace policy considered in this paper are reduced to Brown and Proschan [7] repair model and the modified age-dependent policy of Fontenot and Proschan [10], respectively.

The following situation illustrates the proposed age replacement policy.

In a maintenance organization, there are k repair personnel who have different repair skills. That is, repair personnel have different probabilities of repairing perfectly a failed system depending on their skills. Hence it is quiet natural that the probability of perfect repair of a failed system is randomly distributed.

Consider failure and repair process without planned replacement. Let $N(t)$ be the number of failures in $(0,t)$. Then $N(t)$ is composed of the processes $L(t)$ and $M(t)$ where $L(t)$ is the number of perfect repairs in $(0,t)$ and $M(t)$ is the number of minimal repairs in $(0,t)$.

Let $Y_1=\{t\geq 0\,|\,L(t)=1\}$ be the waiting time at which the first perfect repair occurs and let $Z_1=\{t\geq 0\,|\,M(t)=1\}$ be the waiting time at which the first minimal repair occurs. Then $\overline{H}(t)=P(Y_1\geq t)=\sum_{i=1}^{k}\overline{F}^{p_i}(t)\pi_i$ and $\overline{G}(t)=P(Z_1\geq t)=\sum_{i=1}^{k}\overline{F}^{1-p_i}(t)\pi_i$ (see Lim et al. [13] for more details). And $M(Y_1)$ represents the number of minimal repairs in $(0, Y_1]$. It is noted from Sheu et. al. [21] that Y_1 is independent of $\{M(t), t\geq 0\}$.

Let $r_H(t)$ and $r_G(t)$ be the failure rate functions of H and G, respectively. Here, $r_H(t)$ and $r_G(t)$ are the same as those in Lim et al. [13], but are discretely defined in this paper as follows.

$$r_H(t)=r(t)\frac{A(t,1)}{A(t,0)} \text{ and } r_G(t)=r(t)\frac{Z(t,1)}{Z(t,0)}, \tag{1}$$

where $A(t,n)=\sum_{i=1}^{k}p_i^n\overline{F}^{p_i}(t)\pi_i$ and $Z(t,n)=\sum_{i=1}^{k}(1-p_i)^n\overline{F}^{(1-p_i)}(t)\pi_i$.

Then it is well known that $\{L(t), t\geq 0\}$ and $\{M(t), t\geq 0\}$ are two independent NHPP (Nonhomogeneous Poisson Process) with intensity functions $r_H(t)$ and $r_G(t)$, respectively. Also note that $N(t)=M(t)+L(t)$.

3. Expected cost rate per unit time and optimal replacement schedule

In this section, we derive the mathematical formulas of the expected cost rate per unit time for the proposed age replacement policy for both the infinite-horizon and the one-replacement-cycle cases. And for each case, we investigate the optimal age T^* for replacement which minimizes the expected cost rate per unit time.

3.1. The infinite-horizon case

3.1.1. Expected cost rate per unit time

Let $Y_1, Y_2,\cdots$ be i.i.d. random variables from $H(y)$. Let $Y_i^*=\min(Y_i,T)$ for $i=1,2,\ldots$. Then Y_i^* represents time duration between two successive renewals of system. Renewal means either perfect repair or replacement. Let R_i^* be total cost incurred over the renewal interval Y_i^* for $i=1,2,\ldots$. Y_i^* and R_i^* can be described in more detail as follows.

$$Y_i^*=Y_i\cdot I_{(0,T)}(Y_i)+T\cdot I_{(T,\infty)}(Y_i) \tag{2}$$

and

$$R_i^*=[C_p+C_mM(Y_i)]\cdot I_{(0,T)}(Y_i)+[C_r+C_mM(T)]\cdot I_{(T,\infty)}(Y_i). \tag{3}$$

Then $\{(Y_i^*,R_i^*)\}$ constitutes a renewal reward process. Let $K(t)$ denote the expected cost for operating the system over the time interval $[0,t]$. Then it is well known from the Renewal Reward Theorem (see Proposition 7.3 in Ross [14] that expected long-run cost per unit time, $B(T)$, becomes

$$B(T)=\lim_{t\to\infty}\frac{K(t)}{t}=\frac{E(R_1^*)}{E(Y_1^*)}, \tag{4}$$

where

$$E(Y_1^*)=I_{(0,T)}(y_1)\cdot E(Y_1)+I_{(T,\infty)}(y_1)\cdot T=\int_0^T t\,dH(t)+T\overline{H}(T)$$
$$=\int_0^T\overline{H}(t)dt \tag{5}$$

and

$$E(R_i^*)=C_mEE[I_{(0,T)}(Y_1)\cdot M(Y_1)]+C_pP(Y_1<T)+[C_r+C_mM(T)]P(Y_1\geq T)$$
$$=C_m\int_0^T\overline{H}(t)r_G(t)dt+C_pH(T)+C_r\overline{H}(T). \tag{6}$$