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A condition-based maintenance policy for multi-component systems with Lévy copulas dependence

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ABSTRACT

In this paper, we propose a new condition-based maintenance policy for multi-component systems taking into account stochastic and economic dependences. The stochastic dependence between components due to common environment is modelled by Lévy copulas. Its influence on the maintenance optimization is investigated with different dependence degrees. On the issue of economic dependence providing opportunities to group maintenance activities, a new maintenance decision rule is proposed which permits maintenance grouping. In order to evaluate the performance of the proposed maintenance policy, we compare it to the classical maintenance policies.

1. Introduction

Maintenance optimization is for the purpose of guaranteeing the productivity, ensuring the safety as well as saving the maintenance cost. During the last few decades, maintenance policies have been widely developed for single-unit systems subjected to various degradation models, e.g. Gamma process [1,2], Wiener process [3] and Markov chains [4], to name a few. Some recent works take account for the impact of the system environment [5,6] or heterogeneity among components regarding their parameters within a population sharing the same degradation model [7,8]. However, the results of single-unit systems can be hardly extended to multi-component systems, since the existence of various types of dependences (economic, stochastic and structural [9]) affects the global optimization of system.

The economic dependence provides opportunities to group maintenance actions which can reduce the maintenance cost. Several papers have proposed maintenance grouping strategies to take advantage of it [10–14]. In [10], a dynamic grouping method for age-based replacement with rolling horizon is developed and the component deterioration information is incorporated at decision time. This method is further developed in [11] on infinite horizon considering different levels and combinations of dependences between components. In [12], a dynamic grouping maintenance strategy based on failure rate distribution is proposed where they take the preventive maintenance duration into consideration. In [13,14], the complex system structure is then considered. Generally, these literatures group maintenance activities by four-step approaches: individual optimization, tentative planning, group optimization and update. It is noteworthy that in the aforementioned papers, the inspection cost is neglected when minimizing the cost criterion though the deterioration information is updated through inspection. However, when the inspection becomes significant, it is hard to apply those maintenance grouping strategies to the condition-based maintenance policy. The reason is twofold. On one hand, the optimal date of replacement of component depends on its monitored deterioration level and inspection period such that the individual optimization is not obvious. On the other hand, it is difficult to establish penalty functions whereas in previous literatures the penalty functions plays an important role in the group optimization. To this end, we propose a new condition-based maintenance grouping policy based on a decision rule which permits the maintenance grouping by using the available individual deterioration level of components.

As components work in common environment and share the stress, it is not appropriate to assume that components are independent. Stochastic dependence occurs when the failure of one component influences other components or when components are correlated. Several models of stochastic dependence have been developed in the literature. The failure interactions model where the failure of a component can increase the failure rates of other components is investigated in [15–17]. However, the amount of increase of failure rates caused by the breakdown of a component

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is unobservable and may be hard to estimate in practice [18,19]. Modelled by a bivariate wear subordinator with gamma marginal processes, a preventive maintenance policy is proposed in [20]. According to the bivariate wear subordinator model, components are correlated by a common stochastic deterioration process. However, this model is limited both with regard to the dependence structure and the range of dependence degree between components. Copulas have been widely used in finance to deal with the multivariate distribution function model [21]. On one hand, they can separate the dependence structure from marginal distributions. On the other hand, they provide a wide range of dependence structures and lift the restriction of bivariate wear subordinator where the marginal process must be the same type (e.g. for a two-unit system, one component is Gamma process and the other one is inverse Gaussian process). In [22], ordinary copulas are chosen to describe the stochastic dependence where the dependence structure between components depends on time. To overcome this drawback, Tankov extends the ordinary copulas to Lévy copulas dedicated to Lévy processes in [23] as a time-invariant solution. In addition to this, the possibility to describe diversiform stochastic dependence is another reason that we profit from the Lévy copulas in this paper.

The aim of this paper is to propose a maintenance grouping strategy for condition-based maintenance optimization considering the economic and stochastic dependence simultaneously. The impact of both types of dependence on the maintenance optimization is studied through numerical experiments. The paper is organized as follows. Section 2 is devoted to the description of individual deterioration and stochastic dependence model. Section 3 describes the new condition-based maintenance policy and the implemented classic maintenance policies. In Section 4, numerical experiments are presented and sensitivity analysis is carried out. Conclusions and perspectives are drawn in Section 5.

2. Degradation model and system structure

Throughout the paper, we consider a parallel system with two deteriorating components. The state of system is characterized by the combination of individual deterioration level of each component. The failure behavior of component can be described by a stochastic model which consists of two parts: the individual deterioration models of components and the stochastic dependence modelling. The component fails when its deterioration level exceeds a predetermined corrective threshold L. When both components are in failure state, the system is out of service.

2.1. Individual deterioration model

Note that the Gamma process is time-homogeneous with independent and positive increments so it is sensible to use this process to describe a gradual deterioration [2]. Moreover, the Gamma process has an explicit probability distribution function which permits both the mathematical developments and simulation. When no maintenance action is performed, the behavior of component i is modelled by an increasing deterioration process $(X_t^i)_{t\geq 0}$ $(i=1,2)$ since the damage accumulation is usually irreversible for most systems. Suppose that $(X_t^1)_{t\geq 0}$ and $(X_t^2)_{t\geq 0}$ are two Gamma processes with parameters (α_1, β_1) and (α_2, β_2) respectively such that, for $i=1,2$:

- $X_0^i = 0$,
- $(X_t^i)_{t\geq 0}$ has independent increments,
- For $t>0$ and $h>0$, $X_{t+h}^i - X_h^i$ follows a Gamma distribution with shape parameter $\alpha_i t$ and scale parameter β_i and the probability density function of $(X_{t+h}^i - X_h^i)$ is given by:

$$f(x|\alpha_i t, \beta_i) = \frac{\beta_i^{\alpha_i t} x^{\alpha_i t-1}\exp(-\beta_i x)}{\Gamma(\alpha_i t)}. \tag{1}$$

Parameters α_i et β_i are estimated from the deterioration data and they allow us to model different deterioration behaviors of components in the system.

To simulate a stochastic process, approximations are often used. The Gamma process $(X_t)_{t\geq 0}$ is a specific Lévy process with Lévy measure $\nu(dx) = \frac{\alpha e^{-\beta x}}{x}dx$. Among the four series representations proposed by Rosínski in [24] for Lévy process, the Inverse Lévy Measure Method has been used in this paper for the purpose of being combined with Lévy copulas. Let $U(x) = \int_x^\infty \nu(dt)$ be the tail integral of Lévy measure ν, the inverse of U is defined as:

$$U^{-1}(y) = inf\{x>0, U([x,\infty[) < y\} = \frac{1}{\beta}E_1^{-1}\left(\frac{y}{\alpha}\right), \tag{2}$$

where $y>0$ and $E_1(x) = \int_x^\infty \frac{e^{-u}}{u}du$ is the exponential-integral function. The approximation of the Gamma process $(X_t)_{t\geq 0}$ on $[0,T]$ using a Poisson point process is detailed as follows:

$$X_t = \sum_{n=1}^{\infty} U^{-1}(\Gamma_n/T)\mathbb{1}_{[0,t]}(v_n), \tag{3}$$

where $t\in[0,T]$, $\{\Gamma_n\}_{n\in\mathbb{N}}$ is a sequence of arrival times of a standard Poisson process and $\{v_n\}_{n\in\mathbb{N}}$ is a sequence of independent uniformly distributed random variables on [0,T]. Moreover, $\{\Gamma_n\}_{n\in\mathbb{N}}$ and $\{v_n\}_{n\in\mathbb{N}}$ are independent. In fact, the random variables $\{v_n\}_{n\in\mathbb{N}}$ are related to the arrival times of process $(X_t)_{t\geq 0}$ whereas $\{\Gamma_n\}_{n\in\mathbb{N}}$ are related to the jump sizes (the greater is Γ_n and the smaller is the corresponding jump).

Fig. 1 illustrates the three steps of sampling a Gamma process path on [0,1]. Fig. 1(a) represents a sample of a standard Poisson process $(\Gamma_n)_{n\in\mathbb{N}}$ and the corresponding jump size of the derived Gamma process is depicted, obtained as $E_1^{-1}(\Gamma_n/\alpha)$. It should be noticed that the range of sample for $(\Gamma_n)_{n\in\mathbb{N}}$ ends at $n=17$, because the jump size becomes negligible after that. Fig. 1 (b) represents a sequence of uniform random variables on [0,1] and the corresponding jump at each date is drawn. Finally, Fig. 1 (c) represents the corresponding sample of Gamma process derived from the combination of the two previous samples.

Hence, the final series representation of the Gamma process $(X_t)_{t\geq 0}$ is:

$$X_t = \sum_{n=1}^{\infty} E_1^{-1}(\Gamma_n/\alpha)1_{[0,t]}(v_n)/\beta. \tag{4}$$

Despite the lack of the explicit expression of the inverse exponential-integral function, several methods are available for approximation. For example, when the degree of freedom of χ^2-tail probabilities tends to zero, its survival function tends to the exponential-integral function. Therefore, we adopt the inverse χ^2-tail function in Matlab for simulation [25].

2.2. Modelling stochastic dependence with Lévy copulas

Let $(X_t^1, X_t^2)_{t\geq 0}$ be a bivariate Lévy process describing the two-unit system state with Lévy measure ν, then its tail integral of Lévy measure is:

$$U(x_1, x_2) = \nu([x_1,\infty]\times[x_2,\infty]).$$

As with ordinary copulas, Tankov [23] extends the Sklar's theorem for Lévy process: if U is a tail integral of Lévy measure with margins $U_1(x_1, 0)$, $U_2(0, x_2)$ then there exists a Lévy copula $\mathfrak{C}$ such that:

$$U(x_1, x_2) = \mathfrak{C}(U_1(x_1, 0), U_2(0, x_2)). \tag{5}$$