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Remaining useful lifetime estimation and noisy gamma deterioration process

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ABSTRACT

In many industrial issues where safety, reliability, and availability are considered of first importance, the lifetime prediction is a basic requirement. In this paper, by developing a prognostic probabilistic approach, a remaining lifetime distribution is associated to the system or component under consideration. More particularly, the system's deterioration is modelled by a non-homogeneous gamma process. The model considers a noisy observed degradation data and by using the Gibbs sampling technique, the hidden degradation states are approximated and afterwards the system's remaining useful lifetime distribution is estimated. Our proposed prognosis method is applied to the Prognostic and Health Management (PHM) 2008 conference challenge data and the interest of our probabilistic model is highlighted. To point out the interest of the prognostic, a maintenance decision rule based on the remaining lifetime estimation results is proposed.

1. Introduction

Prognosis and lifetime prediction have been largely developed in theory and industry. The quantity of interest is often the Remaining Useful Lifetime (RUL) and it is estimated based on the degradation measures. These measures often provide information such as the failure time data, failure sensors measurements to assess the reliability of systems. The variety and number of sources of uncertainty (*e.g.* forecast, complex system, unknown degradation process) encourage a probabilistic approach to model the degradation phenomenon. The probabilistic tools which permits the deterioration modelling are the stochastic processes. In many cases such as accumulation of damage in a mechanical systems or the deterioration due to defective products in case of a production line or due to corrosion/erosion in a structural systems, the problem is to deal with a system with monotone increasing deterioration. The appropriate deterioration process to model such deteriorations is the Gamma process because the paths of this latter are discontinuous and they can be thought of as the accumulation of an infinite number of small shocks which it is often how the degradation occurs, refer to [1,2]. Another interest of this process is that it allows feasible mathematical developments. A survey on the Gamma process and its application in degradation modelling is given in [2]. The Gamma process is largely used in deterioration modelling (eg. [3–7]).

In practice, the available data on the health of a system are often noisy due to the presence of different sensors or measurement errors. Usually the sensor measurements on a monotone deteriorating system are contaminated by noises which can totally conceal the monotonicity of the deterioration process. To model such observations and not to neglect the monotonicity of the real deterioration phenomenon in this paper we propose to use a noisy gamma process where the gamma distributed deteriorating states are contaminated with Gaussian noise. This process gather both the noisy observations and the monotone deterioration phenomenon. To deal with this process first the observations are filtered to separate the noise and the deterioration states. Once we have access to the monotonically deteriorating states, we can make life time prediction based on the gamma process properties and the corresponding feasible mathematical calculations. In this framework, it is possible to give a probability distribution for the RUL and hence to obtain a confidence interval for its estimation and measure the precision of our prognosis method.

As a case study for the deterioration model, we consider the degradation indicator obtained from the 2008 Prognostic Health Management Challenge (PHM) data [8] and explored in [9]. The proposed indicator in [9] has a monotonous trend with a large noise. Therefore it seems sensible to model the deterioration indicator evolution by a noisy stochastic process. In this aim a non-homogeneous Gamma process with a Gaussian noise is used to model the deterioration. Hence, a stochastic filtering is

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theoretically proposed to find the hidden degradation states of the gamma process. Gamma process with additive Gaussian noise has been already considered in the literature, refer to [10,11]. In both papers, the parameter estimation through the maximum likelihood calculation in the presence of unobserved deterioration is carried out by numerical approximations. Authors in [10] consider a Monte Carlo method whereas in Lu et al. [11] use the Quasi Monte Carlo method to reduce the calculation time. The Quasi Monte Carlo method proposed in [11] approximates the likelihood function by using the Genz transform. In comparison with [10] the proposed Quasi Monte Carlo method leads to a smaller error and reduces the computation time.

In a general framework to estimate the non-observable system state, the stochastic filtering approaches are frequently used, see [12,13]. The stochastic filtering approaches give an estimation of the system state based on all collected measures history, therefore these methods can give reliable performances for the degradation states estimation. In the framework of stochastic filtering methods, the well-known Kalman filter is essentially used on linear models with Gaussian noises. In [14], the authors introduce a RUL estimation model based on Kalman filter for the aircraft engine. In industry, we usually deal with a system with non-linear behavior and non-necessarily Gaussian noisy observations. Following [15], the non-linear systems are considered in many areas of science and engineering. One of most popular non-linear filtering techniques is the particle filtering, a sequential Monte Carlo technique. We can find the application of the particle filtering in different domains such as signal processing [16], biology, biostatistics [17], etc.

In this paper, we propose another Monte Carlo Markov Chain (MCMC) technique, Gibbs sampling, for stochastic filtering that can be applied to non-linear models and non-Gaussian noises. Recently, this technique is used in [18] to model the survival data (semi-competing risks model) via a gamma frailty. Gibbs sampling can be applied to the models with unknown parameters and can estimate stably the target function (unobserved data) based on the observed data. This sampling method which requires heavy calculations give a precise estimation of the real state and also its associated uncertainty. The use of the Gibbs sampling in the framework of the non-homogeneous gamma process is not a trivial task and this method is not applied before in prognosis. In this paper the use of Gibbs sampling technique is to extract the Gamma degradation states from noisy degradation indicator.

The likelihood function is complex due to the hidden degradation states, therefore the estimation of the model parameters by maximizing the likelihood directly is infeasible. As proposed in [19], the stochastic expectation–maximization (SEM) algorithm is matching to solve this problem. The algorithm can iteratively take the approximated values of hidden data to the complete likelihood function and then estimate parametric inference of model. After the states estimation step, based on the filtered data a probability distribution is associated to the remaining useful life distribution.

In previous works on the same data set (PHM data) a Wiener process was proposed to model the deterioration [9]. In the case of the Wiener process it is supposed that the large variability of data is not due to the presence of noise but to the properties of the system under consideration. Since rarely deteriorating systems have a non-monotone tendency this modeling does not remain valid for a big range of observation data. Another weakness of the use of Wiener process in the deterioration modeling is the unfeasible probability and mathematical calculations which lead us to solve our prognosis problem by Monte Carlo simulations and the prognosis is based on the mean value of the RUL (see [9]). In this paper we propose a more efficient and precise prognosis method which is accordingly more time consuming. The preliminary results of this method are presented in [20,21]. In this paper once the RUL is estimated a predictive maintenance is proposed to prevent system's failure and take advantage of the prognosis procedure.

The remainder of the paper is organized as follows. Section 2 presents the deterioration model using a non-homogeneous gamma process with Gaussian noise and how to filter the hidden degradation states. In section 3 a method for the remaining useful life estimation is proposed and the impact of the observations vectors size on the RUL estimation is studied. Section 4 is devoted to a case study obtained from the 2008 Prognostic Health Management Challenge data. In Section 5, a maintenance decision rule based on the remaining useful life estimation is proposed. Finally, the conclusions of the results as well as the furthers works are given in Section 6.

2. Deterioration modelling and state estimation

2.1. Deterioration model

Let us consider a gradually deteriorating system and we suppose that a scalar random variable can summarize the system state at time t. The system is supposed to have a monotone non-decreasing degradation which is the behavior observed in many physical deterioration processes. Denote by $X(t)$ the system state at time t which is monotone non-decreasing. A survey on Gamma process given in [2] pointed out this process to model a monotone increasing deterioration. The paths of the Gamma process are discontinuous and represent the accumulation of an infinite number of small shocks which is often how the degradation occurs, refer to [1,2].

Let $X(t)$ be a gamma distributed random variable with a scale parameter $\beta > 0$ and shape function $v(t) = \alpha t^b$ which is a non-decreasing, right continuous, real-valued function for $t > 0$ with $\alpha > 0, X(t) \sim \Gamma(v(t), \beta)$. The probability density function is given by:

$$f(x|v,\beta) = \frac{\beta^{v(t)}}{\Gamma(v(t))} x^{v(t)-1} \exp(-\beta x) \mathbb{1}_{[0,+\infty[}(x),$$

where $\Gamma(x) = \int_{z=0}^{\infty} z^{x-1} e^{-z}\, dz$ is the Euler gamma function and $\mathbb{1}_{[a,b]}(x) = 1$ if $x \in [a,b]$ and $\mathbb{1}_{[a,b]}(x) = 0$ otherwise. The gamma process with shape function $v(t) > 0$ and scale parameter β has the following properties:

- $X(0) = 0$,
- $X(s) - X(t) \sim \Gamma(v(s) - v(t), \beta)$ for all $s > t \geq 0$,
- $X(t)$ has independent increments.

It is supposed that the deterioration level is not directly observable. Let $Y_j = Y(t_j),\ j = 1, \ldots, n$, be the degradation indicators observed at inspection times $0 < t_1 < \ldots < t_n$, and $X_j = X(t_j),\ j = 1, \ldots, n$ is the non-observable state at time t_j modelled by a non-homogeneous gamma process, thus the relation between X_j and Y_j can be expressed as follows:

$$Y_j = f(X_j, \epsilon_j) = X_j + \epsilon_j$$

where ϵ_j are independent Gaussian random variables with standard deviation σ_j and mean equal to zero and it can be also expressed as a function of X_j and Y_j as follows: $\epsilon_j = g(X_j, Y_j) = Y_j - X_j$.

For an efficient prognosis and maintenance planning it is necessary to estimate the non-observable state of the system. In this aim, the hidden degradation states vector $\mathbf{X} = (X_1, \ldots, X_n)$ should be evaluated from the observations $\mathbf{Y} = (Y_1, \ldots, Y_n)$. Due to uncertainties related to the vector $\mathbf{X}$ and to the observations $\mathbf{Y}$, to propose a relevant estimation of $\mathbf{X}$ and to be able to analyse the estimation performances we should first calculate the conditional