Optimal loading and protection of multi-state systems considering performance sharing mechanism

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ARTICLE INFO

Article history:
Received 29 April 2015
Received in revised form 19 September 2015
Accepted 7 December 2015
Available online 17 December 2015

Keywords:
Multi-state systems
Performance sharing
Load dependent failure
Protection
Universal generating function
Genetic algorithms

ABSTRACT

Engineering systems are designed to carry the load. The performance of the system largely depends on how much load it carries. On the other hand, the failure rate of the system is strongly affected by its load. Besides internal failures, such as fatigue and aging process, systems may also fail due to external impacts such as nature disasters and terrorism. In this paper, we integrate the effect of loading and protection of external impacts on multi-state systems with performance sharing mechanism. The objective of this research is to determine how to balance the load and protection on system elements. An availability evaluation algorithm of the proposed system is suggested and the corresponding optimization problem is solved utilizing genetic algorithms.

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1. Introduction

A multi-state series-parallel system consists of N sub-systems connected in series and each sub-system contains several multi-state elements connected in parallel [1–6]. The system fails if and only if at least one of the sub-systems cannot meet its required demand. In the literature, most of the works assume that each sub-system satisfies its own demand individually. However, many examples in real situations such as power systems, communication systems and data processing systems indicate that the surplus performance from one sub-system can be transmitted to other sub-systems that are experiencing the performance deficiency. This type of performance sharing mechanism was first studied in [7], and extended in [8,9].

Many research works have considered the element allocation and maintenance of the multi-state series-parallel system [5,10,11], but very limited work considered the effect of loading on the system elements. However, a majority of the engineering systems are designed to carry the load, such as coal conveyors, cargo trucks, and power generating units. The performance of such systems depends on the amount of load it is carrying. Besides, many studies have empirically shown that the failure rate of the system element is strongly affected by its working load [12,13]. Therefore, it is important to consider the effect of loading when analyzing the availability of multi-state systems. As a result, several recent research works have studied the optimal loading of multi-state systems [14–19].

Besides the internal failures such as aging process and fatigue, the system element may also fail due to external impacts such as nature disasters and terrorism [20]. One approach for improving the survivability of system elements is to make defensive investment to protect the elements [21–23]. The probability that an element is destroyed by the external impact is usually modeled as a function of the external impacts intensity and the protection effort allocated on the element [24–27]. The survival probability of an element is higher if more protection effort is allocated onto it. The optimal trade-off between investment into the maintenance and protection of the elements in a simple parallel system that is subject to both internal failure and natural disasters was studied in [27].

In this paper, we consider a multi-state series-parallel system with common bus performance sharing as shown in Fig. 1. The performance of each sub-system is the cumulative performance of the elements within the sub-system. If the performance of any sub-system is more than its demand, the surplus performance can be transmitted to any other deficient sub-system via the common bus. However, the total amount of the performance that can be transmitted among different sub-systems is subject to the capacity of the common bus. In this research, the capacity is also assumed to be a random variable since the common bus may be
2. Load dependent failure rate and performance

Every multi-state element \( e_i \) considered in this paper is able to carry load of different values. However, element \( e_i \) can only be in two states: failure state with zero performance and functioning state with working performance \( g_i(L_i) \), where \( L_i \) is the load carried by element \( e_i \) and \( g_i(L_i) \) which denotes the performance of element \( e_i \) as a function of load \( L_i \). The load on element \( e_i \) can vary from \( L_{i, \min} \) to \( L_{i, \max} \), where \( L_{i, \min} \) and \( L_{i, \max} \) denote the minimum and maximum allowable load on element \( e_i \) respectively. This is the reason why element \( e_i \) is considered to be multi-state. As discussed earlier, the expected performance of element \( e_i \) is not a monotonic function of the load \( L_i \).

It can be difficult to distinguish the load and the performance in some situations since they can be considered the same. For example, the pressure on a pipe which carries fluid in a laminar flow mode is proportional to the volume of the flow, i.e., the throughput (performance) is proportional to the pressure (load). However, the load-performance relationship can be nonlinear. For instance, consider a pipe carrying fluid in a turbulent flow mode. The pressure on the pipe is a non-linear function of volume of the flow.

The load dependent performance function \( g_i(L) \) can take different expressions depending on the situations. Without loss of generality, the function measuring relationship between the performance and the load can be assumed as follows.

\[
g_i(L_i) = q_i + c_i L_i \tag{1}
\]

where \( q_i \) and \( c_i \) are the coefficients of the linear equation.

To analyze the availability of systems made up by elements with load dependent failure rates, the load failure relationship must be known beforehand since the availability of each element can be derived based on its failure rate. In the literature, the accelerated life test models play an important role in determining the load-failure rate relationships. The existing accelerated life test models are summarized in [28]. In this paper, the commonly used proportional hazard model (PHM) will be discussed and used in the numerical experiments.

PHM, which was first proposed in [29], has received popularity in the field of reliability engineering in recent years [30–32]. PHM states that the failure rate of an element is the product of the baseline hazard rate and factors based on the conditions. In general, PHM can be expressed as follows.

\[
h(t|X) = h_0(t) \cdot e^{\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_q X_q} \tag{2}
\]

where \( h_0(t) \) denotes the baseline hazard rate as a function of time. \( X_1, \ldots, X_q \) are the factors that affect the hazard rate function and \( \beta_1, \ldots, \beta_q \) are the corresponding coefficients.