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Model-reduction techniques for reliability-based design problems of complex structural systems

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ABSTRACT

This work presents a strategy for dealing with reliability-based design problems of a class of linear and nonlinear finite element models under stochastic excitation. In general, the solution of this class of problems is computationally very demanding due to the large number of finite element model analyses required during the design process. A model reduction technique combined with an appropriate optimization scheme is proposed to carry out the design process efficiently in a reduced space of generalized coordinates. In particular, a method based on component mode synthesis is implemented to define a reduced-order model for the structural system. The re-analyses of the component or substructure modes as well as the re-assembling of the reduced-order system matrices due to changes in the values of the design variables are avoided. The effectiveness of the proposed model reduction technique in the context of reliability-based design problems is demonstrated by two numerical examples.

1. Introduction

Structural design via deterministic mathematical programming techniques has been widely accepted as a viable tool for engineering design [1]. However, in most structural engineering applications response predictions are based on models involving uncertain parameters. This is due to a lack of information about the value of system parameters external to the structure such as environmental loads or internal such as system behavior. Under uncertain conditions the field of reliability-based optimization provides a realistic and rational framework for structural optimization which explicitly accounts for the uncertainties [2–4]. In the present work, structural design problems involving finite element models under stochastic loading are considered. The design problem is formulated as the minimization of an objective function subject to multiple design requirements including standard and reliability constraints. The probability that any response of interest exceeds in magnitude some specified threshold level within a given time duration is used to characterize the system reliability. This probability is commonly known as the first excursion probability [5]. The corresponding reliability problem is expressed in terms of a multidimensional probability integral involving a large number of uncertain parameters. Reliability-based design formulations require advanced and efficient tools for structural modeling, reliability analysis and mathematical programming. Modeling and analysis techniques of structural systems are well established and sufficiently well documented in the literature [6]. On the other hand, several tools for assessing structural reliability have lately experienced a substantial development providing solution of involved systems [7–9]. In the field of reliability-based optimization of stochastic dynamical systems several procedures have been recently developed allowing the solution of problems dealing with finite element models of relatively small number of degrees of freedom [10–14]. However, the application of reliability-based optimization to stochastic dynamical systems involving medium/large finite element models remains somewhat limited. In fact, the solution of reliability-based design problems of stochastic finite element models requires a large number of finite element analyses to be perform during the design process. These analyses correspond to finite element re-analyses over the design space (required by the optimizer), and system responses over the uncertain parameter space (required by the simulation technique for reliability estimation). Consequently, the computational demands depend highly on the number of finite element analyses and the time taken for performing an individual finite element analysis. Thus, the computational demands in solving reliability-based design problems may be large or even excessive.

In this context, it is the main objective of this work to present a framework for integrating a model reduction technique into the reliability-based design formulation of a class of stochastic linear and nonlinear finite element models. The goal is to reduce the time consuming operations involved in the re-analyses and

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dynamic responses of medium/large finite element models. Specifically, a model reduction technique based on substructure coupling for dynamic analysis is considered in the present implementation [15]. The proposed method corresponds to a generalization of substructure coupling applicable to systems with localized nonlinearities. The technique includes dividing the linear components of the structural system into a number of substructures obtaining reduced-order models of the substructures, and then assembling a reduced-order model for the entire structure. In summary, the novel aspect of this contribution involves a strategy for integrating a model reduction technique into the reliability-based design formulation of medium/large finite element models under stochastic excitation. This represents an additional area of application of substructure coupling which has been already used for uncertainty management in structural dynamics with applications in areas such as uncertainty analysis, finite element model updating, and reliability sensitivity analysis [16–19]. The organization of this work is as follows. The formulation of the reliability-based design problem is presented in Section 2. Next, the characterization of the structural systems of interest is considered in Section 3. Implementation issues such as reliability estimation, optimization strategy and model reduction are discussed in Section 4. The mathematical background of the model reduction technique is outlined in Section 5. The integration of the model reduction technique into the design process is discussed in Section 6. The effectiveness of the proposed strategy is demonstrated in Section 7 by the reliability-based design of two structural systems. The paper closes with some conclusions and final remarks.

2. Problem formulation

The reliability-based design problem is characterized in terms of the following constrained non-linear optimization problem

$$\begin{aligned}&\text{Min}_{\boldsymbol{\theta}} \quad C(\boldsymbol{\theta})\\ &\quad \text{s.t.} \quad g_i(\boldsymbol{\theta}) \le 0 \qquad i=1,\ldots,n_c\\ &\qquad P_{F_i}(\boldsymbol{\theta}) - P^*_{F_i} \le 0\,, \quad i=1,\ldots,n_r\\ &\qquad \boldsymbol{\theta} \in \Theta\end{aligned} \tag{1}$$

where $\boldsymbol{\theta}, \theta_i, i=1,\ldots,n_d$ is the vector of design variables with side constraints $\theta_i^l \le \theta_i \le \theta_i^u$, $C(\boldsymbol{\theta})$ is the objective function, $g_i(\boldsymbol{\theta}) \le 0, i=1,\ldots,n_c$ are standard constraints, and $P_{F_i}(\boldsymbol{\theta}) - P^*_{F_i} \le 0$ are the reliability constraints which are defined in terms of the failure probability functions $P_{F_i}(\boldsymbol{\theta})$ and target failure probabilities $P^*_{F_i}$, $i=1,\ldots,n_r$. It is assumed that the objective and constraint functions are smooth functions of the design variables. The objective function $C(\boldsymbol{\theta})$ can be defined in terms of initial, construction, repair or downtime costs, structural weight, or general cost functions. The standard constraints are related to general design requirements such as geometric conditions, material cost components, and availability of materials. On the other hand, the reliability constraints are associated with design specifications characterized through the use of reliability measures given in terms of failure probabilities with respect to specific failure criteria. For structural systems under stochastic excitation the probability that design conditions are satisfied within a particular reference period T provides a useful reliability measure [5]. Such measure is referred as the first excursion probability and quantifies the plausibility of the occurrence of unacceptable behavior (failure) of the structural system. In this context, a failure event F_i can be defined as $F_i(\boldsymbol{\theta}, \mathbf{z}) = d_i(\boldsymbol{\theta}, \mathbf{z}) > 1$, where d_i is the so-called normalized demand function defined as $d_i(\boldsymbol{\theta}, \mathbf{z}) = \max_{j=1,\ldots,l} \max_{t \in [0,T]} |r_j^i(t, \boldsymbol{\theta}, \mathbf{z})| / r_j^{i^*}$, where $\mathbf{z} \in \Omega_{\mathbf{z}} \subset R^{n_z}$ is the vector of uncertain variables involved in the problem (characterization of the excitation), $r_j^i(t, \boldsymbol{\theta}, \mathbf{z}), j=1,\ldots,l$ are the response functions associated with the failure event F_i, and $r_j^{i^*}$ is the acceptable response level for the response r_j^i. It is clear that the responses $r_j^i(t, \boldsymbol{\theta}, \mathbf{z})$ are functions of time (due to the dynamic nature of the excitation), the design vector $\boldsymbol{\theta}$, and the random vector $\mathbf{z}$. These response functions are obtained from the solution of the equation of motion that characterizes the structural model (see next Section). The uncertain variables $\mathbf{z}$ are modeled using a prescribed probability density function $p(\mathbf{z})$. This function indicates the relative plausibility of the possible values of the uncertain parameters $\mathbf{z} \in \Omega_{\mathbf{z}}$. The probability of failure evaluated at the design $\boldsymbol{\theta}$ is formally defined as

$$P_{F_i}(\boldsymbol{\theta}) = P\left[\max_{j=1,\ldots,l} \max_{t \in [0,T]} \frac{|r_j^i(t, \boldsymbol{\theta}, \mathbf{z})|}{r_j^{i^*}} > 1\right] \tag{2}$$

where $P[\cdot]$ is the probability that the expression in parenthesis is true. Equivalently, the failure probability function evaluated at the design $\boldsymbol{\theta}$ can be written in terms of the multidimensional probability integral

$$P_{F_i}(\boldsymbol{\theta}) = \int_{d_i(\boldsymbol{\theta}, \mathbf{z}) > 1} p(\mathbf{z})\, d\mathbf{z} \tag{3}$$

It is noted that the above formulation can be extended in a direct manner if the cost of partial or total failure consequences is also included in the definition of the objective function. It is also noted that constraints related to statistics of structural responses (i.e. mean value and/or higher-order statistical moments) can be included in the formulation as well. Thus, the above formulation is quite general in the sense that different reliability-based optimization formulations can be considered.

3. Mechanical modeling

A quite general class of structural dynamical systems can be cast into the following equation of motion

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{k}(\mathbf{u}(t), \dot{\mathbf{u}}(t), \boldsymbol{\tau}(t)) + \mathbf{f}(t) \tag{4}$$

where $\mathbf{u}(t)$ denotes the displacement vector of dimension n, $\dot{\mathbf{u}}(t)$ the velocity vector, $\ddot{\mathbf{u}}(t)$ the acceleration vector, $\mathbf{k}(\mathbf{u}(t), \dot{\mathbf{u}}(t), \boldsymbol{\tau}(t))$ the vector of non-linear restoring forces, $\boldsymbol{\tau}(t)$ the vector of a set of variables which describes the state of the nonlinear components, and $\mathbf{f}(t)$ the external force vector. The matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ describe the mass, damping, and stiffness, respectively. Note that some of the matrices and vectors involved in the equation of motion depend on the vector of design variables $\boldsymbol{\theta}$ and/or the uncertain system parameters $\mathbf{z}$ and therefore the solution is also a function of these quantities. The explicit dependence of the response on these quantities is not shown here for simplicity in notation. The evolution of the set of variables $\boldsymbol{\tau}(t)$ is described by a first-order differential equation

$$\dot{\boldsymbol{\tau}}(t) = \boldsymbol{\kappa}(\mathbf{u}(t), \dot{\mathbf{u}}(t), \boldsymbol{\tau}(t)) \tag{5}$$

where $\boldsymbol{\kappa}$ represents a non-linear vector function. This characterization allows to model different types of nonlinearities including hysteresis and degradation [20,21]. From Eq. (5) it is seen that the set of variables $\boldsymbol{\tau}(t)$ is a function of the displacements $\mathbf{u}(t)$ and the velocities $\dot{\mathbf{u}}(t)$, i.e. $\boldsymbol{\tau}(\mathbf{u}(t), \dot{\mathbf{u}}(t))$. Therefore, Eqs. (4) and (5) constitute a system of coupled non-linear differential equations for $\mathbf{u}(t)$ and $\boldsymbol{\tau}(t)$. The previous formulation is particularly well suited for cases where most of the components of the structural system remain linear and only a small part behaves in a nonlinear manner. Such cases are of particular interest in the present work, that is, linear finite element models with localized nonlinearities. The external force vector $\mathbf{f}(t)$ is modeled as a non-stationary stochastic process. Depending on the application under consideration different methodologies are available for generating these types of processes. Such methodologies include filtered Gaussian white noise processes, stochastic processes compatible with power spectral densities, point source-based models, and record-based