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Modelling and forecasting the trends of life cycle curves in the production of non-renewable resources

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ABSTRACT

In this study, we review mathematical models of trends in the production of non-renewable resources. We propose new models that allow specifying curve asymmetry and use genetic algorithms as curve-fitting methods. We estimate the quality of fit of our proposed models and use them to predict oil production in the OECD (Organisation for Economic Co-operation and Development) countries, the EU, the U.S., Norway, Syria, the UK, and some fields in Russia; gas production in the EU, the UK and Italy; shale gas production in the U.S.; coal production in the U.S. and Germany; as well as global gold production. The models fit the data accurately in all these cases.

1. Introduction

Presently, we model oil and gas production life cycles using simulation as well as curve-fitting models [1]. Modelling the extraction of non-renewable resources reveals that simulation models are limited to certain spheres of application.

Researchers note [2] considerable subjectivity in primary geological data interpolation. As a result, systematic errors often occur when using simulation models. It is stated [1] that simulation models are subject to poor data availability or discrepancies in data. The problem of geological data collection and organization becomes crucial when field level data is aggregated at regional, national, or global levels. The simulation approach does not describe field characteristics. It does not take into account the financial capacity of the operator, and ignores seasonal effects and human factors. Moreover, it does not consider market demand dynamics.

Curve-fitting models seem to be more suitable for extraction analysis at different levels of aggregation. We use the curve-fitting approach to model non-renewable resources extraction $Y(t)$ over time$t$.

2. Models and methods

2.1. The most popular extraction models

Curve-fitting models of oil production have been in use since the 1950s [1,3]. Most of them use bell-shaped curves to describe the stages of growth and decline during the production life cycle. They were initially used to model oil and gas extraction, but can be also applied to other non-renewable resources. Deffeyes [4] identifies three primary curve-fitting models.

1. The Hubbert model [5,6]:

$$Y(t) = \frac{Y_{max} \cdot 2}{1 + \cosh(\sigma(t - t_0))} + \varepsilon(t), \qquad (1)$$

where $\varepsilon(t)$ is the residual, t_0 is the time point when the production peak Y_{max} is reached, and$\sigma()$ is the parameter defining the model peak width.

The Hubbert model is used by many researchers [7–13].

2. The Cauchy distribution (Lorenz) model [4,14]:

$$Y(t) = \frac{Y_{max} \cdot \sigma^2}{(t - t_0)^2 + \sigma^2} + \varepsilon(t). \qquad (2)$$

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3. The Gaussian bell-shaped model [9,15,16]:

$$Y(t) = Y_{max}e^{-(t-t_0)^2/\sigma^2} + \varepsilon(t). \quad (3)$$

In addition, the SNPP (skewed-normal production-profile) model has become popular [13,17–20]. It is described by.

$$Y(t) = At^n e^{-\alpha t} + \varepsilon(t), \quad (4)$$

where n, α are the model parameters.

The lognormal distribution model [21,22] is given by.

$$Y(t) = \frac{A}{t}e^{-\frac{[\ln(t)-b]^2}{\sigma^2}} + \varepsilon(t). \quad (5)$$

Sometimes researchers also employ multi-function (piecewise) models. The certain intervals are modelled using exponential or polynomial functions forming a bell-shaped curve [16,23]. However, it is difficult to predict the disruption points. In addition, with these models, accurate modelling of resource depletion after the production peak requires a large sample size. Thus, such models have poor predictive power as regards the stage of decline, which is a stage of primary interest. Owing to this shortcoming, we do not consider multi-function models further.

2.2. *The complex of extraction models with specified asymmetry*

To compare models (1)–(5), it is convenient to normalize them according to the peak coordinates and the slope of the bell-shaped curve. The Gaussian bell curve is used as a standard of normalization, where σ is the standard deviation of the curve.

Then the models are defined as follows:

-Hubbert

$$Y(t) = \frac{Y_{max}\cdot 2}{1+\cosh\left(\frac{\sqrt{2}(t-t_0)}{\sigma}\right)} + \varepsilon(t); \quad (6)$$

-Cauchy

$$Y(t) = \frac{Y_{max}\cdot 2\sigma^2}{(t-t_0)^2 + 2\sigma^2} + \varepsilon(t); \quad (7)$$

-Gauss

$$Y(t) = Y_{max}e^{-\frac{(t-t_0)^2}{2\sigma^2}} + \varepsilon(t); \quad (8)$$

-SNPP

$$Y(t) = Y_{max}\left(\frac{e}{t_0}\right)^{\frac{t_0^2}{\sigma^2}}\cdot t^{\frac{t_0^2}{\sigma^2}} e^{-\frac{t_0}{\sigma^2}\cdot t} + \varepsilon(t); \quad (9)$$

-Lognormal model

$$Y(t) = Y_{max}\frac{t_0}{t}e^{-\frac{1}{2}\frac{\ln\frac{t}{t_0}\left(t_0^2\cdot\ln\frac{t}{t_0}-2\sigma^2\right)}{\sigma^2}} + \varepsilon(t). \quad (10)$$

In these models, σ determines the slope of the production curve (see Fig. 1a). We see that the Hubbert and the Gauss models look similar, but their inflection points are different. In the Gauss model, the inflection points occur earlier, while the Hubbert curve has a 'wider' top than the Gauss curve (Fig. 1b).

The five curves have significantly different shapes. Thus, the models (6)–(8) are symmetric, while models (9) and (10) are asymmetric.

Although the oil production life cycle curves may be symmetric in some cases [24], the opposite is usually true in reality. For example, Brandt analysed production samples at 67 oil production

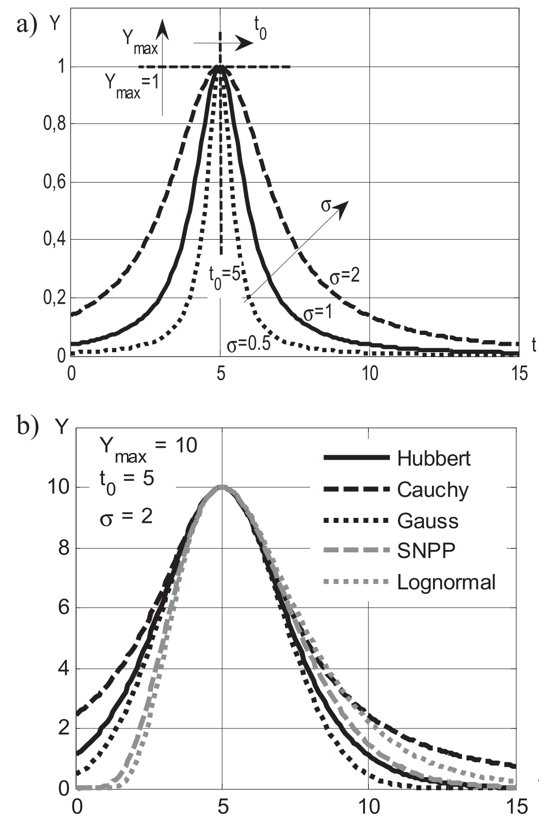

Fig. 1. Historical oil production models: a) changing parameters, b) comparing the shapes of models (6)–(10).

fields [8,16]. He used the Gauss bell-shaped model (3) with a time-varying slope parameter captured by the Verhulst logistic function. This suggestion allowed specifying the asymmetry of most oil production curves based on the longer duration of the decline stage. The asymmetry can be explained by the various decisions (e.g. technological) taken to maintain production at the highest possible level after the peak.

We can expect other production life cycle curves to display asymmetry. This provides motivation for exploring a set of models to represent different types of asymmetry. Out of this set, we can choose the models that fit the real data best and that provide the most accurate forecasts.

For this reason, we suggest modifying the life cycle curves (6)–(10) using other functions to represent the slope parameter σ. This enables us to build new asymmetric models. Then, we can find the combinations of models and asymmetry functions that yield the best results in terms of fit and predictive power.

Setting a constant.

$$\sigma = \sigma_1 \quad (11)$$

gives us a symmetrical production curve.

We assume that the slope parameter σ changes from σ_1 at the growth stage to σ_2 at the stage of decline, in accordance with the following asymmetry functions (which are logistic laws): -Verhulst [16,25]

$$\sigma = \sigma_1 + \frac{\sigma_2-\sigma_1}{1+e^{-\frac{t-t_0}{\sigma_T}}}; \quad (12)$$

-Richards [26,27]

$$\sigma = \sigma_1 + \frac{\sigma_2-\sigma_1}{\left[1+e^{-\frac{t-t_0}{\sigma_T}}\right]^{1/\sigma_{T1}}}; \quad (13)$$