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# Analytical tools for calculating the maximum heat transfer of annular stepped fins with internal heat generation and radiation effects

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ABSTRACT

ASFs (Annular stepped fins) require less material than ADFs (annular disc fins) while retaining the ability to produce the same cooling rate in a convection environment. A simple analysis was developed for ASFs that considered radiative heat transfer and heat generated by a nuclear reactor through linearization of the radiation terms. The linearized equations were solved by exact and approximate analytical methods. Without any linearization, a new closed-form analysis was established for the temperature profile with the help of the differential transform method. An integral differential transform method was introduced to determine the actual heat-transfer rate when heat was generated inside an ASF under nonlinear radiation surface conditions. The temperature results obtained using this analytical approach were compared with those obtained from a finite-difference analysis, and were in excellent agreement. The fin performance was defined as a function of the heat generated for a given set of design conditions. An optimization study with varying heat generation was carried out to compare the performance of ADFs and ASFs, which highlighted the superior aspects of an annular fin design.



## 1. Introduction

Rapid movement of heat is required in a growing number of engineering applications to avoid system overheating and increase the life span of components. Annular fins have numerous applications, including compact heat exchangers, specialized installations of single-and double-pipe heat exchangers, electrical components from which generated heat must be efficiently dissipated, and internal combustion engines cooled by air. In a conventional heat exchanger, heat is transferred from one fluid to another through a metallic wall. The rate of heat transfer is directly proportional to the extent of the wall surface, the heat-transfer coefficient, and the temperature difference between the fluid and the adjacent surface [1].

The basic mechanism of heat transfer through fins is to conduct heat from a heat source via the fins, and then dissipate the heat to the surrounding air by convection, radiation, or simultaneous convection–radiation. In general, thermal convection dominates heat transfer from the fin surface to the surroundings. However, a high fin-surface temperature with respect to the surroundings produces radiative heat transfer that cannot be omitted from the heat-transfer analysis. A fin with heat generation, which will have a high fin surface temperature, could be used to cool nuclear reactors, where heat is generated from a nuclear source consisting of rapidly moving neutrons and gamma rays [2].

To increase the heat-transfer rate from a fluid-carrying tube, annular fins are attached to the outer surface. This is a standard practice to augment the heat transfer from a primary cylindrical surface. However, since the cross-sectional area of annular disc fins is constant, the fin material does not effectively conduct heat near the fin tip. Hence, different tapered profiles (for example, triangular, trapezoidal, parabolic, or hyperbolic) have been proposed in the literature [1]. These profiles make better use of the fin material than a constant-thickness fin while being able to maintain the same heat-transfer rate, but may require complex fabrication processes. Alternatively, an annular disc fin with a step change in thickness both saves material and is easy to fabricate [3].

Annular fins are an important part of fin-and-tube heat exchangers. The primary concern of most investigations of such heat exchangers is the performance of the annular fins. Chambers and Somers [4] determined the performance of an annular fin with a rectangular profile for boundary conditions consisting of a constant temperature at the fin base and insulation at the fin tip. Smith and Sucec [5] calculated the efficiency of triangular fins using a power-

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**Nomenclature**

| | |
|---|---|
| $A_j$ | dimensionless variables defined in Eq. (32b) for $j = 1, 2, 3, 4$ |
| ADF | annular disc fin |
| ASF | annular stepped fin |
| $B_1$, $B_2$ | variables defined by Eqs. (9a) and (9b), respectively |
| $B_3$, $B_4$, $B_5$ | variables defined by Eq. (11) |
| Bi | Biot number, $h\,r_1/k_f$ |
| $\mathrm{Bi}_l$ | Biot number based on the convective heat-transfer coefficient at the fin base, $h_l r_1/k_f$ |
| $D_1$, $D_2$, $D_3$, $D_4$ | operating roots, see Eqs. (19a) and (19b) |
| $E_1$, $E_2$, $E_3$ | parameters defined by Eqs. (19d) and (19e) |
| $F_j$ | variables defined by Eq. (23a) for $j = 1, 2, 3, 4$ |
| $F(m)$, $G(m)$ | differential transform functions |
| $G_1$, $G_2$, $G_3$, $G_4$ | parameters defined by Eq. (23b) |
| $h$ | convective heat-transfer coefficient over the entire fin surface(W m$^{-2}$ K$^{-1}$) |
| $h_l$ | convective heat-transfer coefficient at the fin base surface for convective heating(W m$^{-2}$ K$^{-1}$) |
| $I_m(Z)$ | modified Bessel function of first order $m$ and argument$Z$ |
| $K_m(Z)$ | modified Bessel function of second order $m$ and argument $Z$ |
| $k_f$ | thermal conductivity of the fin material (W m$^{-1}$ K$^{-1}$) |
| $Q$ | dimensionless actual heat-transfer rate, $q/4\pi\,k_f\,r_1(T_b - T_a)$ |
| $Q_i$ | dimensionless ideal heat-transfer rate, $q_i/4\pi\,k_f\,r_1(T_b - T_a)$ |
| $Q_{\mathrm{opt}}$ | optimum or maximum value of $Q$ |
| $Q_0$ | dimensionless heat-transfer rate, $q_0/4\pi\,k_f\,r_1(T_b - T_a)$ |
| $q$ | actual heat-transfer rate through the fin (W) |
| $q'''$ | volumetric heat generation rate as a linear function of temperature (W m$^{-3}$) |
| $q_A$ | dimensionless volumetric heat generation rate at temperature $T_a$, see Eq. (3b) |
| $q_i$ | ideal heat-transfer rate through a fin (W) |
| $q_0$ | heat-transfer rate through the base area in the absence of a fin (W) |
| $R$ | dimensionless radial coordinate, $r/r_3$ |
| $\overline{R}$ | dimensionless mean radius, $\overline{r}/r_3$ |
| $R_1$ | dimensionless inner radius, $r_1/r_3$ |
| $R_2$ | dimensionless step radius, $r_2/r_3$ |
| $r$ | radial coordinate starting from the center of the tube (m) |
| $\overline{r}$ | mean radius (m) |
| $r_1$ | inner radius (m) |
| $r_2$ | step radius of an ASF (m) |
| $r_3$ | outer radius (m) |
| $T$ | local fin surface temperature (K) |
| $T_a$ | surrounding temperature (K) |
| $T_b$ | base temperature (K) |
| $T_i$ | fin surface temperature at the step section (K) |
| $T_l$ | fluid temperature on the fin base surface (K) |
| $T_{\max}$ | maximum fin surface temperature (K) |
| $t_1$ | semi-base thickness (m) |
| $t_2$ | semi-tip thickness (m) |
| $U$ | dimensionless fin volume, $V/2\pi r_1^3$ |
| $V$ | fin volume (m$^3$) |
| $X$ | dimensionless coordinate, $x/r_3$ |
| $x$ | coordinate from the step change section to the base for ASFs (m) |
| $Y$ | dimensionless coordinate, $y/r_3$ |
| $y$ | coordinate from the tip to the step change in thickness for ASFs (m) |
| $Z_A$ | dimensionless variable,$Z_0^2/R_1^2$ |
| $Z_B$ | dimensionless variable,$Z_A/\tau$ |
| $Z_0$ | dimensionless fin parameter, $\sqrt{\mathrm{Bi}/\psi}$ |
| $Z_1$ | dimensionless parameter defined by Eq. (9c) |
| $Z_2$ | parameter defined by Eq. (9d) |

*Greek letters*

| | |
|---|---|
| $\alpha$ | variable heat-generation parameter (K$^{-1}$) |
| $\beta$ | dimensionless variable heat-generation parameter, $\alpha\,(T_b - T_a)$ |
| $\gamma$ | radiative–convective parameter, $4\sigma\varepsilon T_a^3/h$ |
| $\delta$ | dimensionless small length that tends toward zero |
| $\varepsilon$ | emissivity |
| $\varepsilon_f$ | fin effectiveness |
| $\eta_f$ | fin efficiency |
| $\sigma$ | Boltzmann constant (W m$^{-2}$ K$^{-4}$) |
| $\psi$ | dimensionless thickness, $t/r_1$ |
| $\tau$ | thickness ratio, $t_2/t_1$ |
| $\theta$ | dimensionless temperature, $(T - T_a)/(T_b - T_a)$ |
| $\theta_i$ | dimensionless temperature at the step for ASFs, $(T_i - T_a)/(T_b - T_a)$ |
| $\theta_l$ | dimensionless fluid temperature, $(T_l - T_a)/(T_b - T_a)$ |
| $\theta_{max}$ | dimensionless maximum temperature, $(T_{\max} - T_a)/(T_b - T_a)$ |
| $\phi$ | dimensionless temperature − heat-generation parameter, $\theta - q_A/Z_1^2$ |

series solution for the temperature distribution. Sikka and Iqbal [6] adopted a finite-difference procedure to analyze the effectiveness of radiative–convective fins, whereas Sparrow and Niewerth [7] developed a numerical linearized solution. Aziz et al. [8] studied a uniformly thick radial fin with convective heating at the base and convective–radiative cooling at the tip for homogeneous and functionally graded materials, with internal heat generation. The fin was exposed to convection and radiation. Mustafa et al. [9] carried out a thermal analysis of orthotropic annular fins with contact resistance using separation of variables.

In most applications, the optimal fin shape is important because weight and material costs are the primary design considerations. Optimization studies for radial fins with a specified geometry have been carried out by Laor and Kalman [10], Yu and Chen [11], Heggs and Ooi [12], Lai et al. [13], and Aziz [14]. Ullmann and Kalman [15] used a constant heat-transfer coefficient to determine the efficiency of annular fins with various tapered profiles, whereas Kundu and Barman [16] established an analysis based on a Frobenius series expansion to determine the performance and optimum dimensions of annular disc fins under dehumidifying conditions based on linear relationships between the temperature and humidity. Hatami and Ganji [17] described the thermal performance of circular convective–radiative porous fins with different sectional shapes and materials. Least square and Runge–Kutta methods were applied to predict the temperature distribution. Kundu and Lee [18] developed an analytical solution for the heat transfer for different shapes of wet longitudinal fins based on the differential transform method while accounting for all nonlinearity effects. Later, Torabi and Zhang [19] extended this work to convective–radiative environments.

All of the above studies used fins with specific geometries, either with constant or variable fin thickness. To improve the heat-transfer

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