ARTICLE IN PRESS

Energy xxx (2014) 1–10

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

Displacer gap losses in beta and gamma Stirling engines

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ARTICLE INFO

Article history:
Received 6 March 2014
Received in revised form
6 May 2014
Accepted 8 May 2014
Available online xxx

Keywords:
Stirling engine
Displacer gap losses
Shuttle heat transfer
Optimization

ABSTRACT

An analytical model has been developed to evaluate "displacer gap losses" in the clearance between the displacer and the cylinder in both β and γ configurations of Stirling engines. Displacer gap losses are the sum of the "shuttle heat transfer" and the "enthalpy pumping".

The present model takes into account the pressure gradient in the gap, the gas compressibility and real gas effect. Gas velocity and temperature distributions in the displacer, in the gap and in the cylinder were determined by solving momentum and energy balances in a concentric tubes geometry. Our model is then introduced in a whole engine model and the effect of the clearance thickness and engine's speed on total displacer gap losses are investigated. Novel tendencies of the solution are observed and new ways for optimization are demonstrated.

1. Introduction

The Stirling engine has been invented since the 19th century but was unable to penetrate the energy and transport sector despite its flexibility and relatively high thermal efficiency. The use of this technology has been concentrated in cryogenics and spatial applications. However, with the present growth of concerns about energy saving and environmental issues, Stirling engines becomes more interesting than ever. It is the best solution to make micro-scale co-generation since the ORC (organic Rankine cycle) is relatively less efficient for a power level below 100 kW_{el} [1]. Stirling cycle implies that a constant mass of working gas (usually air, helium or hydrogen) alternates between two temperature levels and passes through four thermodynamic processes: an isothermal compression, an isochoric heating, an isothermal expansion and finally an isochoric cooling [2]. In β and γ configurations of Stirling engines (Fig. 1) this reciprocating motion is assured by a displacer. To avoid frictional losses and excessive wear, clearance seals are generally used instead of ring seals. The inconvenience of this solution is that a gas leakage occurs from the compression to the expansion chamber or back depending on the displacer's direction of motion and the pressure difference between these spaces. This leakage causes an enthalpy pumping through the displacer's clearance [3].

Besides, a temperature gradient exists along the cylinder wall. Thus, the displacer is moving between a hot and a cold region and causes heat transfer with its motion. It takes some heat from the hot region of the cylinder and rejects it in the cold region. This amount of lost heat is called "shuttle heat transfer". The sum of enthalpy pumping and shuttle heat transfer loss are known in the litterature as "Dispalcer gap losses" [3].

Urieli [4], used a steady state model (steady displacer velocity and axial pressure gradient) to calculate the leakage mass flow through the displacer's clearance. Urieli's formula was used for several years. Rios [5] developed an approximate solution by linearization and application of Fourier series to calculate the shuttle heat transfer. Later, Baik and Chang [6] developed an analytic solution for shuttle heat transfer by considering only conduction in both cylinder and displacer and by neglecting the effect of the gap flow between them. This effect was introduced later by Chang et al. [7]. But, they considered a simple geometry (parallel plates) and assumed that the gas flow in the gap is only due to the displace motion without any effect of the pressure gradient in the gap. Finally, Kotsubo and Swift [8] applied the thermoacoustic theory to study enthalpy pumping and shuttle heat transfer. They considered a parallel plates geometry and supposed that the leakage flow is *a priori* known.

Some numerical studies exist also like the 1D study proposed by Andersen [9] and the work of Huang and Berggren [10]. These studies have been conducted for specific engines and used a

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http://dx.doi.org/10.1016/j.energy.2014.05.017
0360-5442/

Nomenclature

C_p	specific heat capacity, J K^{-1} kg^{-1}
L	length, m
p	pressure, Pa
Pr	Prandtl number
R	radius, m or perfect gas spec const., J kg^{-1} K^{-1}
r	radial coordinate, m
T	temperature, K
t	time, s
u	velocity, ms^{-1}
V	volume, m^3
x	position, m
z	axial coordinate, m

Greek letters

α	thermal diffusivity, m^2 s^{-1}
β	isochoric thermal pressure coefficient, K^{-1}
Γ	temperature gradient in the gap, Km^{-1}
γ	ratio C_p/C_v
λ	thermal conductivity, W K^{-1} m^{-1}
ν	kinematic viscosity, m^2 s^{-1}
ω	angular speed, rad°s^{-1}

Subscripts

0	time-average
c	cylinder or compression space
d	displacer
e	expansion space
f	fluid
H	high temperature source
h	hot HEX
k	cold HEX
L	low temperature sink
n	nth order of Fourier series

Superscripts

*	complex number
~	oscillating part

convective heat transfer coefficient calculated from correlations. Besides, numeric studies are greedy in terms of computing time and then can not be used in whole engine optimization approaches.

In the present study, efforts are put in modeling analytically displacer gap losses by solving momentum and energy equations in the annular gap between the cylinder and the displacer. Unlike Baik [6] and Chang [7], flow and pressure gradient in the gap are taken into account here and that affects considerably solutions and new phenomena are observed. Introducing our model in a whole engine model, we demonstrate the existence of an optimal clearance thickness and an optimal engine speed that minimizes displacer gap losses.

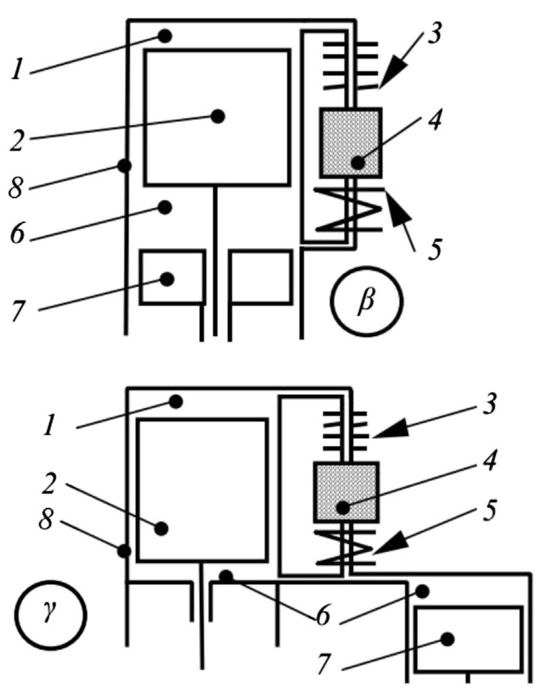

Fig. 1. β and γ configurations of the Stirling engine: 1 – expansion chamber; 2 – displacer; 3 – heater; 4 – regenerator; 5 – cooler; 6 – compression chamber; 7 – power piston; 8 – cylinder.

2. Problem formulation

2.1. Problem description

As shown in Fig. 1, The displacer separates the compression and the expansion spaces. The pressure difference between these spaces is caused by the friction of the gas flowing through the heat exchangers and the regenerator. The problem is modeled in this work as a pulsating flow in the annular gap between two concentric tubes: the displacer and the cylinder. The displacer is moving periodically relative to the cylinder. In Fig. 2, two types of coordinate systems are used: a stationnary system (z,r) for the cylinder and the gap fluid and a moving system (z_d,r_d) for the displacer. These coordinate systems are related by the displacer's motion:

$$r_d = r \tag{1a}$$

$$z_d = z - x_d(t) \tag{1b}$$

where $x_d(t)$ is the axial coordinate of the origin of the moving coordinate system in the stationary one. It represents also the position of the displacer relative to the cylinder.

2.2. Assumptions

As said previously, we keep some assumptions from previous works [6–8]. These are:

1. The fluid flow is laminar and fully developed.
2. The thickness of the cylinder or the displacer is greater than the thermal penetration depth.
3. Radiation heat transfer is neglected.
4. Axial temperature gradients in the displacer, in the cylinder and in the gap are constant and equal to each other. This gradient is approximated by $\Gamma = T_H - T_L/L_d$.

However,

- Contrary to Baik [6] and Chang [7], the gas is not assumed incompressible and pressure gradient effect in the gap is taken