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Predictive inference for system reliability after common-cause component failures

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ABSTRACT

This paper presents nonparametric predictive inference for system reliability following common-cause failures of components. It is assumed that a single failure event may lead to simultaneous failure of multiple components. Data consist of frequencies of such events involving particular numbers of components. These data are used to predict the number of components that will fail at the next failure event. The effect of failure of one or more components on the system reliability is taken into account through the system's survival signature. The predictive performance of the approach, in which uncertainty is quantified using lower and upper probabilities, is analysed with the use of ROC curves. While this approach is presented for a basic scenario of a system consisting of only a single type of components and without consideration of failure behaviour over time, it provides many opportunities for more general modelling and inference, these are briefly discussed together with the related research challenges.

1. Introduction

A major consideration for reliability of systems is the possible occurrence of common-cause failures, where multiple components fail simultaneously due to the same underlying cause. This paper considers the reliability of a system following a future failure event, with possible common-cause failures of multiple components. In particular, the aim is to develop predictive inference for the reliability of a system with multiple components based on previous failure event data for this system, or for other systems that are fully exchangeable with this system. These data are assumed to consist of the numbers of failing components in past failure events, in each of which at least one component failed. These failure events did not necessarily involve failure of the whole system, but each event involves failure of one or more components. Aspects of ageing are not taken into account, nor any other aspects that are explicitly related to time, usage or other processes. Such aspects are likely to be important in some practical applications, developing methodology to deal with these provides interesting research challenges.

Common-cause failures are important in many applications, as systems with built in redundancy are at increased risk if multiple components may fail simultaneously. Basic concepts of modelling common-cause failures are reviewed by Rasmuson and Kelly [1] and by Mosleh et al. [2]. The alpha-factor model, proposed by Mosleh et al. [3] is commonly used, and enables straightforward analysis in the framework of Bayesian statistics due to the availability of conjugate prior distributions [4]. A robust Bayesian approach to this model has recently been proposed by Troffaes et al. [5], using a set of conjugate priors instead of a single prior distribution, following a standard approach for generalized Bayesian methods in theory of imprecise probabilities [6]. In this paper an alternative statistical method within the theory of imprecise probability is used, namely nonparametric predictive inference (NPI) [7]. For the problem considered here, it makes little difference which specific method for statistical inference is used to predict the number of components failing simultaneously at the next common-cause failure event. Due to its explicitly predictive nature it is attractive to use the NPI approach. The main contribution of the paper is the link from inference on this number of failing components to lower and upper probabilities for the event that the system will still function after the next common-cause failure event. This link is quite straightforward, as shown in this paper, with the use of the system survival signature [8], a recently introduced concept which is closely related to the popular system signature [9] but, in contrast to the latter, is also conceptually straightforward for systems with multiple types of components. The question of the system's reliability following a common-cause

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failure event is of practical interest, as following such an event one may have more time to consider appropriate maintenance or replacement actions if the system is still functioning than in case the system has failed. Such further actions are not addressed in this paper, but combined with aspects of failure processes they provide interesting topics for future research.

As mentioned, the main contribution of this paper is to model the link between failure of components and failure of the system, or indeed functioning of the system, for the common-cause failure scenario, through the recently introduced concept of survival signatures [8]. For a system with m components, the state vector $\underline{x}=(x_1,x_2,\ldots,x_m)\in\{0,1\}^m$ is defined such that $x_i=1$ if the ith component functions and $x_i=0$ if not, using an arbitrary but fixed labelling of the components. The structure function $\phi:\{0,1\}^m\rightarrow\{0,1\}$, defined for all possible $\underline{x}$, takes the value 1 if the system functions and 0 if the system does not function for the state vector $\underline{x}$. The structure function can be generalized by defining it as the probability that the system functions for the state vector $\underline{x}$, which might be relevant if one is not certain about its functioning [10], this is not used in this paper but provides interesting challenges for research and opportunities for applications.

In this paper, attention is restricted to coherent systems, which means that $\phi(\underline{x})$ is not decreasing in any of the components of $\underline{x}$, so system functioning can never be improved by worse performance of one or more of its components. It is further assumed that $\phi(\underline{0})=0$ and $\phi(\underline{1})=1$, so the system fails if all its components fail and it functions if all its components function. These assumptions could easily be relaxed but they simplify presentation in this paper and are reasonable for most systems of practical interest. More importantly, attention is restricted in this paper to systems consisting of exchangeable components, which could be called components of a single type. Most practical systems, in particular also networks, consist of components of multiple types. This restriction is merely for sake of simplicity of presentation, the method introduced here can quite straightforwardly be generalized to systems with multiple types of components.

The survival signature for a system with m exchangeable components, denoted by $\Phi(l)$, for $l=1,\ldots,m$, is the probability that the system functions given that precisely l of its components function [8]. For coherent systems, $\Phi(l)$ is an increasing function of l, and the second assumption above leads to $\Phi(0)=0$ and $\Phi(m)=1$. There are $\binom{m}{l}$ state vectors $\underline{x}$ with precisely l components $x_i=1$, so with $\sum_{i=1}^m x_i=l$; let S_l denote the set of these state vectors. Due to the exchangeability assumption for the m components, or more precisely the assumed exchangeability of the random quantities representing functioning of the m components, all these state vectors are equally likely to occur, hence

$$\Phi(l)=\binom{m}{l}^{-1}\sum_{\underline{x}\in S_l}\phi(\underline{x}) \tag{1}$$

Coolen and Coolen-Maturi [8] called $\Phi(l)$ the survival signature because, by its definition, it is closely related to survival of the system, and it is close in nature to the system signature [9]. The survival signature can straightforwardly be generalized to systems with multiple types of components, in contrast to the system signatures for which this is practically impossible [8].

This paper is organised as follows. Section 2 presents the use of nonparametric predictive inference to predict the number of failing components at a future common-cause failure event. In Section 3 these inferences are combined with the survival signature to lead to lower and upper probabilities for the event that the system will still function after the next failure event. As with any newly proposed procedure, it is important to evaluate the performance of the presented method, this is non-trivial for methods using lower and upper probabilities. In Section 4 a novel way to evaluate the performance of such an imprecise probability method is introduced, which is fully in line with the predictive nature of the inferences and makes use of ROC curves. Section 5 concludes the paper with a discussion of the method and related research challenges.

2. Predicting the number of failing components

For a system with m exchangeable components, which are assumed throughout this paper to all function before the failure event of interest, a common-cause failure event can lead to simultaneous failure of any number $f\in\{1,\ldots,m\}$ of components. The alpha-factor model [2,3] introduces parameters α_j, for $j=1,\ldots,m$, representing the probability that precisely j of the m components in the system fail simultaneously, given that a failure event occurs, hence $\sum_{j=1}^m\alpha_j=1$. It is possible to select values α_j based on background knowledge of the system, but in this paper we aim at learning about these probabilities from available failure data while attempting to add only rather minimal further assumptions. Recently, two Bayesian solutions to the same problem have been presented, one using a non-informative Dirichlet prior distribution [4] and the other using the imprecise Dirichlet model (IDM) [5]. Both of these let the numbers 1 to m of possible simultaneous failures be represented by m categories, with an assumed multinomial distribution for the numbers of observations in these categories. This is a standard statistical approach, with the IDM the most widely applied imprecise probability model in statistics [6,11] with also interesting applications in reliability, see for example [12,13]. There are, however, two issues with this model. Even while the (set of) prior distribution(s) is chosen in order to have little influence on the inferences, they do affect these noticeably in case interest is in categories which have rarely or even never been observed. Furthermore, any ordering of the categories is not taken into account. The latter aspect is particularly relevant if one is interested in events involving unions of categories, for example that the number of simultaneously failing components is at least two.

This paper presents a nonparametric predictive inference (NPI) alternative to the combination of the alpha-factor model with (imprecise) Dirichlet prior(s), also using lower and upper probabilities to quantify uncertainty. NPI for multinomial data has been presented as an alternative to the IDM [14], while NPI for real-valued data, including right-censored observations, has also been applied successfully [15,16]. However, in this paper attention is restricted to NPI for ordinal data [17] as this explicitly takes the natural ordering of the m categories, namely the numbers $1,\ldots,m$, into account. Assume that data are available on n previous common-cause failure events, which are assumed to be exchangeable with the next event, implying that they also all involved m components, and all components, in the observed systems and for the prediction, are exchangeable. Let $n_j\geq 0$ denote the number of the observed common-cause events in which precisely j components failed simultaneously, so $\sum_{j=1}^m n_j=n$. It is important to emphasize that no information or assumption is used about which specific components failed, just the numbers of components failing at the failure events are of interest for the inferences considered in this paper.

It is important to comment briefly on the assumption of exchangeability of all components, both in the observed systems and for prediction. For simplicity, one can consider this assumption as being in line with assuming all components have the same (unknown) probability of failure in case of a common-cause failure event. Hence, one could not, for example, apply the method presented in this paper in a scenario where components that have been in a system for a longer time undergo some wear, in the sense of their probability of failure increasing. If one would deem