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Analytical method for optimization of maintenance policy based on available system failure data

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ABSTRACT

An analytical optimization method for preventive maintenance (PM) policy with minimal repair at failure, periodic maintenance, and replacement is proposed for systems with historical failure time data influenced by a current PM policy. The method includes a new imperfect PM model based on Weibull distribution and incorporates the current maintenance interval T_0 and the optimal maintenance interval T to be found. The Weibull parameters are analytically estimated using maximum likelihood estimation. Based on this model, the optimal number of PM and the optimal maintenance interval for minimizing the expected cost over an infinite time horizon are also analytically determined. A number of examples are presented involving different failure time data and current maintenance intervals to analyze how the proposed analytical optimization method for periodic PM policy performances in response to changes in the distribution of the failure data and the current maintenance interval.

1. Introduction

Maintenance involves preventive and corrective actions carried out to keep physical systems in the desired operating condition or to restore them to this condition. Optimal maintenance policies aim to provide optimal system reliability/availability and safety performance at lowest possible maintenance costs. The literature on maintenance is vast. For a full overview on the state-of-the-art, the readers are referred to see [1–6].

Maintenance can be categorized into three groups: (1) corrective maintenance (CM), (2) preventive maintenance (PM) and (3) predictive maintenance (PdM). CM are actions performed when the system fails. The most common form of CM is "minimal repair", where the state of the system after repair is nearly the same as that just before failure (see [7,8]). PM is a maintenance policy based on replacing, overhauling or remanufacturing a system at fixed or adaptive time intervals, regardless of its condition at the time. The periodic PM policy can be considered as the most common maintenance policy in which a system is preventively maintained at fixed time intervals, regardless of the failure history of the system; [9–12]. PdM is an advanced preventive approach where maintenance is deferred until it is actually needed. The objective of this approach is to monitor the system in order to detect incipient faults before they can cause a part to fail [13]. This maintenance strategy has been implemented as condition based maintenance in systems where certain performance indices are periodically or continuously monitored [14–16].

PM policy has been considered by many researchers as one of the most studied maintenance policies (see [17–20]). For most industrial plants, PM is still a dominant maintenance policy as it is easy to implement and not many systems can be condition-monitored [21]. A more comprehensive definition is: PM policy is a planned maintenance that reduces or eliminates accumulated system deterioration, and is executed according with planned schedules. In the reliability and maintenance literature, PM policies are commonly classified as [22]: periodic and sequential PM.

Periodic PM is executed at integer multiples of some fixed time interval. On the other hand, sequential PM is implemented at intervals of unequal time lengths. Sequential PM is more suitable when the system requires more frequent maintenance as it ages, whereas periodic PM is more convenient to schedule. This paper addresses the problem of optimal periodic PM policy for systems with minimal repairs at failures between PM actions and replacements.

In the periodic PM policy, a system receives PM at fixed time intervals kT ($k=1, 2, \ldots, N$), where T is the time interval between PM actions, and is replaced at the Nth PM action. It is assumed that the system receives only minimal repairs at failures occurring between PM actions, and hence, the system failure rate remains unchanged [23].

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The aim of periodic PM optimization is to determine the optimal maintenance interval T^* and the optimal number of maintenance actions N^*, such that the total mean cost of repairs, PM, and replacement activities is minimal. Let us consider one cycle between two adjacent replacements with a constant time NT. From the renewal theorem [24], the expected cost per unit of time for an infinite time span is

$$C(N,T) = \lim_{t\to\infty} \frac{\hat{C}(t)}{t} = \frac{\textit{Expected cost in one cycle}}{\textit{Mean time of one cycle}}. \quad (1)$$

Therefore, the expected cost per unit of time for the periodic PM policy is given by [23]

$$C(N,T) = \frac{1}{NT}\left((N-1)C_{PM} + C_R + C_{MR}\sum_{k=1}^{N}\int_0^T \rho_k(t)\,dt\right) \quad (2)$$

where C_{PM} is the cost of PM actions, C_{MR} is the cost of minimal repair, C_R is the cost of replacement, where $C_R \ge C_{PM}$, and $\rho_k(t)$ is the hazard rate function that describes the impact of a kth PM action at time t. The optimal T^* and N^* have been estimated traditionally by numerical techniques, since no analytical method has been proposed in the literature [22,25–27].

In general, the impact of PM actions can be classified into one of the following situations [25]: perfect, minimal, and imperfect. A perfect PM restores the system to the state "as good as new". A minimal PM restores the system to the state that it was just before the maintenance action, or "as bad as old". An imperfect PM takes the system to any state between "as good as new" and "as bad as old". In practice, PM is usually imperfect.

Imperfect PM has grown recently as a popular issue to researchers as well as industrial applications (see for example [26,28–31]). In order to model the impact of imperfect PM, the hazard rate function of the system under maintenance is generally used. In fact, the hazard rate usually is more informative about the underlying mechanism of failure than the other representatives of a lifetime distribution. For this reason, consideration of the hazard rate may be the dominant method for modeling imperfect PM.

Most of the hazard rate used in imperfect PM models are based on univariate analysis, where the single random variable under analysis is the failure time [32]. Recently, several attempts have been made to extend the concept of the univariate hazard rate to the multivariate analysis in order to include variables that influence the failure time of the system under study, for example cumulative load applied, time varying stress, and environmental factors [33,31]. However, the hazard rate concept is somewhat difficult to extend to the multivariate situation and frequently the observed lifetime data set is not big enough in relation to the dimension of the hazard rate model in order to find good estimates of the model and its parameters. Therefore, this paper is devoted to imperfect PM models based on univariate analysis with the failure time as the random variable under study.

A number of PM models have been developed in order to describe the impact of imperfect PM on the hazard rate of repairable systems. These PM models can be classified into three groups [25]: age reduction models, hazard rate models, and hybrids of both. Age reduction models assume that there is an effective age reduction right after a PM action, and that the hazard rate continues to be a function of the effective age [34–36]. In other words, if T_i and $\rho_i(t)$ represent the PM interval and the hazard rate function of the system prior to the ith PM action, respectively, then the hazard rate function after the ith PM action becomes $\rho_i(t+a_iT_i)$ for $t\in\{\overline{0,T_{i+1}}\}$, where $0<a_i<1$ is the age reduction factor.

The hazard rate models assume that right after a PM action, the hazard rate reduces to zero, and then increases faster than it did in the previous PM interval [37,35,36,38]. Thus, the hazard rate function becomes $b_i\rho_i(t)$ for $t\in\{\overline{0,T_{i+1}}\}$ after the ith PM action, where $b_i>1$ is the hazard rate increase factor. In the hybrid models, the hazard rate becomes $b_i\rho_i(t+a_iT_i)$ after the ith PM action [39,40].

The above models have made important contributions to this research field, however, in practice the main problem is how to take decisions or make inferences about these unknown age reduction and hazard rate increase factors. Numerous approaches have been proposed based on guessing the values of these factors by subjective means, which is fine, as long as there is enough expert knowledge to perform this task properly. Other approaches are based on estimating these factors from observed data. These statistical inference techniques are very good if there are sufficient data to estimate the factors accurately. However, in practice, few data are available in many areas of maintenance and replacement [21].

In addition to the aforementioned issue, generally a PM model requires to estimate the parameters of the lifetime distribution used to determine the hazard rate function on which the model is based. The two-parameter Weibull distribution is one of the most popular distributions for modeling stochastic deterioration of systems because it is very flexible, and can model many types of failure rate behaviors through an appropriate choice of parameters. The estimation of these parameters has been addressed in the literature by various techniques, such as probability plotting, moment estimation, modified moment estimation and maximum likelihood estimation (MLE). In the case of MLEs, the corresponding likelihood equations need to be solved numerically and related software programs need to be applied [41]. Moreover, since the solution is numerical, issues of existence and uniqueness of the estimates have to be addressed, which gets quite involved in the case of scarce data [42].

In this paper, an analytical method for optimizing the periodic PM policy is proposed. This method includes a new hazard rate function based on the two-parameter Weibull distribution in order to assess the impact of imperfect PM actions on the reliability of repairable systems. The proposed hazard rate function does not require adjustment factors as the ones presented in the literature since it is formulated as a function of the both, the current maintenance interval T_0 and the optimal maintenance interval T^* to be estimated. The maintenance interval T_0 refers to the PM policy that is currently influencing the system failure times behavior. Also, this paper proposes an analytical method to estimate the Weibull shape β and scale α parameters that define the hazard rate function. By using the MLE method, a closed-form expression is obtained for the β parameter. The closed-form expression for the α parameter is a function of the β parameter, and it is obtained directly from the partial derivative of the log-likelihood function. Finally, with the proposed hazard rate function, the optimal number of PM actions N^* and the optimal PM interval T^*, both of which minimize the expected total cost function (2), are also analytically estimated.

The paper is organized as follows: the new proposed hazard rate function is described in Section 2. In Section 3 the development of an analytical method to estimate the two Weibull parameters α and β is presented. An analytical approach to estimate the optimal periodic PM policy is proposed in Section 4. In Section 5 some examples of application of the proposed PM optimization method are presented. Finally, some concluding remarks are given in Section 6.

2. The hazard rate function

Let us consider that a system undergoes imperfect periodic PM at fixed time intervals kT, and the time of the kth PM action is $t_k = kT$, $k=\{\overline{1,N}\}$. At the time t_N, the PM action is the replacement