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Efficient analysis of multi-state *k*-out-of-*n* systems

Yuchang Mo [a,*], Liudong Xing [b], Suprasad V. Amari [c], Joanne Bechta Dugan [d]

[a] Department of Computer Science & Technology, Zhejiang Normal University, China
[b] Department of Electrical & Computer Engineering, University of Massachusetts, Dartmouth, USA
[c] Parametric Technology Corporation, USA
[d] Department of Electrical & Computer Engineering, University of Virginia, Charlottesville, USA

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ABSTRACT

Many practical systems are multi-state *k*-out-of-*n* systems with independent, non-identical components, where the system and its components have multiple performance levels and maybe multiple failure modes. Furthermore the system may have different requirements on the number of working components (i.e., value of *k*) for different system state levels. This paper proposes a new analytical method based on multi-valued decision diagrams (MDDs) for the reliability analysis of such multi-state *k*-out-of-*n* systems. MDDs have recently been applied to the reliability analysis of general multi-state systems (MSS). In this work, we make the new contribution by proposing a novel and efficient algorithm for constructing the system MDD that is designed to fully make use of the well-defined *k*-out-of-*n* structure. Examples show how the MDD models are generated using the proposed algorithm, and are then evaluated to obtain the system reliability measures. Performance of the MDD-based method is compared with that of an existing recursive algorithm through a comprehensive benchmark study. Empirical results show that the proposed MDD-based method can offer lower computational complexity than the recursive algorithms, and it can be effectively applied to large practical cases for multi-state *k*-out-of-*n* systems.

1. Introduction

The *k*-out-of-*n* system structure is a commonly used redundancy technique and has a wide range of applications in aerospace, nuclear power, airborne weapon systems, computing and communication systems [1]. Considerable research efforts have been expended in the reliability evaluation of binary *k*-out-of-*n* systems with identical or non-identical components [2–4].

Many practical systems and their components have more than two states (i.e., operational and failed). On the system level, multiple states can be interpreted as multiple levels of system capacity or performance. On the component level, the multiple states can be interpreted as different performance levels and also multiple failure modes with each mode having a different impact on the system level performance. These systems are modeled as multi-state systems (MSS) [5,6]. Consider a power generating system consisting of several independent and identical power generators [6]. The abilities of the system to meet high, normal, and low power load demand can be regarded as different system states. At the component (i.e., generator) level, different types of failures can cause the entire power generating system to completely fail or work at some reduced capacity.

Representative research efforts on the reliability evaluation of multi-state *k*-out-of-*n* systems with independently and identically distributed (i.i.d.) components are [7–11]. Among these works, the algorithm proposed by Huang et al. [7] is enumerating in nature, and therefore it is not efficient. In addition, this enumerating method is applicable only when the ***k*** vector consisting of the *k* values with respect to all system states follows a monotonic pattern (decreasing or increasing). Compared with the enumerating method in [7], the recursive algorithm proposed by Zuo and Tian [8] is more efficient and general (applicable for both monotonic and non-monotonic ***k*** vector). Zuo and Tian's approach is still enumerating in nature, and requires the storage and performance of calculations considering *k* values and component state probabilities. Hence, Zuo and Tian's algorithm may require a huge amount of memory and a considerable amount of time for updating the vectors. As a result, Zuo and Tian's algorithm becomes very slow, even for moderate sized systems. Compared with the algorithms in [7,8], the conditional probability-based algorithm proposed in [9] is not enumerating and does not require storing vectors of intermediate results. Thus it is efficient and can evaluate the reliability and related performance measures of large multi-state *k*-out-of-*n* systems with i.i.d. components in a short time.

* Corresponding author.
E-mail addresses: myc@zjnu.cn (Y.C. Mo), lxing@umassd.edu (L. Xing), samari@ptc.com (S.V. Amari), jbd@virginia.edu (J. Bechta Dugan).

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Nomenclature

n	number of components in the system
M	maximum state level of a multi-state system and its components
k_i	the k value with respect to level i of a generalized multi-state *k-out-of-n* system
$\boldsymbol{k}$	the $\boldsymbol{k}$ vector of a generalized multi-state k-out-of-n system, $\boldsymbol{k}=(k_1, k_2,\ldots, k_M)$
$\boldsymbol{x}$	vector of component state $(x_1, x_2,\ldots, x_N)$
$\varphi(\boldsymbol{x})$	system state structure function, $0 \le \varphi(\boldsymbol{x}) \le M$
E, F	decision diagrams
$size(E)$	number of nodes in decision diagram E

Acronyms

BDD	binary decision diagram
MDD	multiple-valued decision diagrams
MSS	multi-state system
i.i.d.	independently and identically distributed

Research efforts have been expended for more general and practical multi-state k-out-of-n system with independent components where different components typically have different state distributions. Zhao and Cui proposed a reliability evaluation method based on FMCI approach [10]. However, Zhao and Cui's method still suffer from computational inefficiency on systems with large number of components and states. Recently, Tian et al. developed a recursive method for the reliability evaluation of generalized multi-state k-out-of-n systems with independent but non-identical components [11]. This recursive method in [11] involves transforming the system into a nominal decreasing multi-state k-out-of-n system and a recursive algorithm that calculates the probability of this transformed system in the highest nominal state, which gives the probability that the multi-state k-out-of-n systems is in state j or above. However, experimental data shows that the calculation time of the recursive algorithm in [11] can significantly increase as the system size increases, which makes it inefficient or even impractical for analyzing large-scale multi-state k-out-of-n systems with non-identical components.

In this paper, we develop a multiple-valued decision diagrams (MDD)-based method, called "MDD-k/n" to cope with the performance problem of the recursive algorithm in [11]. Benchmark study is performed to show that the proposed MDD-k/n method can offer more efficient reliability calculations, and analyze large-scale multi-state k-out-of-n systems more quickly than the existing recursive method.

The remainder of this paper is organized as follows: Section 2 introduces some background knowledge of generalized multi-state k-out-of-n systems and MDD. Section 3 describes an illustrative example. Section 4 is devoted to the development of the new method MDD-k/n for system analysis. Section 5 gives an illustration of our method through detailed analyses of the example generalized multi-state k-out-of-n system. Section 6 presents further studies on a benchmark to illustrate the advantages of the proposed MDD-based method. Section 7 adapts the proposed MDD-based method for the evaluation of other types of multi-state k-out-of-n system models proposed in literature. Finally, in Section 8, we present our conclusions, as well as directions for future work.

2. Preliminary concepts

2.1. System model

The first multi-state k-out-of-n system model was proposed by El-Neweihi et al [12]. In this model, the system state was defined as the state of the kth best component. i.e., for the system to be in a particular state j or above, there should be at least k components in state j or above. Boedigheimer and Kapur proposed their definition from the perspective of lower and upper boundary points [13], which was actually consistent with the multi-state k-out-of-n model of [12]. For both definitions, the k value is the same with respect to all the system states.

The increased understanding of system behavior has led to the creation of new definitions and evaluation algorithms. These new definitions and algorithms can be used to describe and analyze the complex practical systems found in modern systems. For example, the state-dependent k values can be used to specify state-dependent failure or success criteria. Such generalized multi-state system has important applications in phased-mission systems [11], where the multi-state definition of a component is based on the number of consecutive phases it has completed successfully. This can lead to the situation where the system is able to meet the requirements of a higher level state but unable to meet the requirements of some lower level states. Consider a multi-engine aircraft used in a phased-mission system, where the last phase of taxi-in usually needs only one engine whereas the previous phases (e.g., take-off and level-flight) usually require more than one engine.

To allow different k values with respect to different states, a generalized multi-state k-out-of-n system model was proposed as follows [11]: "An n-component system is called a k-out-of-n: G system if $\varphi(\mathbf{x}) \ge j (1 \le j \le M)$ whenever at least k_l components are in state l or above for all l such that $1 \le l \le j$." Based on this definition, to be in state j or above, the system has to meet all the requirements on the number of components at states from one to j. Thus, the system states in this model are ordered, i.e., if the system is in state j it is also considered to be in state i $(1 \le i \le j \le M)$.

As stated in [11], this model implies multi-state monotone systems: (i) the system state is non-decreasing with the increase of each component state; (ii) the system is in state 0 if all of its components are in state 0, and the system is in state M (the highest possible state) if all of its components are in state M. Many practical engineering systems can fit into the model of [11]; refer to Section 3 for a specific example. In this paper we focus on this general model.

2.2. MDD

To analyze a multi-state system, dependencies among different states of the same component must be considered. Traditionally, these dependencies are accounted for in system reliability analysis using methods such as universal generating functions [14], binary decision diagrams [15], and their generalizations [16]. However, the reliability analysis of these systems remains a challenging task for the general case due to its inherent complexity. Recently, an efficient algorithm based on MDD has been proposed to analyze dynamic systems [17] and phased mission system [18].

MDD are efficient graph-based data structures for symbolic representation and manipulation of discrete multi-valued logical functions [19,20]. Based on Shannon's decomposition theorem, they can represent logical functions as rooted, directed acyclic