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Thermoacoustic oscillation basing on parameter exciting

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A B S T R A C T

To reveal the physical mechanism of the thermoacoustic oscillation is an important task of basic research on thermoacoustics. The startup process of a thermoacoustic oscillation is a parametric excitation process. The so-called "parameter exciting" refers to that the oscillation process is realized by the periodic change of parameter in the system. It is generated by the capacitance change in a real thermoacoustic system. In this paper, the thermoacoustic system is simulated as a nonlinear fluid network. It consists of a variable capacitor, an inductor, a nonlinear resistor and a pumping source. It is in line with an equation of nonlinear parametric vibration. The equation is deduced by following mass, momentum, energy and state equations of the working gas parcel simultaneously. The solution of the equation of nonlinear parametric vibration has been obtained using "averaging method". The analysis shows that the inductance–capacitance resonance for the thermoacoustic system is one of the parameter oscillations with a limit cycle. The condition of a homeostatic periodic motion for the thermoacoustic system has been discussed. The stability of the thermoacoustic oscillation described by the limit cycle has been analyzed using "Lyapunov function method". This theory gives a better understanding of the mechanism of thermoacoustic oscillation.

1. Introduction

The effort by humans to explain nature dates to ancient times. Taoism, which originated in ancient China, can reveal the secret of nature. Tao is a function of two variables, Yin and Yang [1]. Here, Yang and Yin are corresponding to heaven and earth, or rigidity and flexibility, respectively. "Yin–Yang equilibrium" or "a combination of rigidity and flexibility" means that a system (or things) running well in order. For a fluid system, flow inductance corresponding to the rigidity of Taoism represents the flow inertia of the fluid, and flow capacitance corresponding to the flexibility of Taoism denotes the elasticity of the fluid. The combination of rigidity and flexibility in a fluid system is resonance caused by the inductance and the capacitance.

Winning hard hearts with soft words (or conquering the unyielding with the yielding) was advocated by philosophers in ancient China. The resonance in a fluid system can be caused by coupling the inductance with the capacitance through the change of capacitance in the flow. This, in an electric circuit, is called varactor parametric excitation [2].

Thermoacoustic engines (including prime movers and refrigerators) are a new class of energy conversion devices without mechanical moving parts [3–8]. It converts between thermal energy and acoustic power utilizing the thermoacoustic effect [9]. In a thermoacoustic stack with a forced temperature gradient, which is an important part of thermoacoustic engine, the interactions between the entropy wave and the oscillating flow produce a rich variety of thermoacoustic phenomena. The potential application of thermoacoustic engines has driven theoretical and experimental developments [10–14]. For long time extensive studies have been conducted in order to provide a thorough understanding of the thermoacoustic oscillation. Remarkable progress such as the network model [15], mechanism of parameter exciting [16–18], investigation on characteristic time of system [18], thermodynamic optimization [19–22], simulation using lattice gas model [23], and nonlinear thermoacoustic theory [24,25] have been made to theoretically explain the thermoacoustic effects.

Although the performance of a thermoacoustic engine may be calculated based on existent theory, the onset characteristics of thermoacoustic effect are still poorly understood. How is the random heat energy converted into the ordered acoustic energy

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with stable frequency, stable phase angle and stable amplitude? Is the thermoacoustic oscillation in a thermoacoustic stack stable? These issues should be studied in order to promote the development of thermoacoustics. Li Q et al. [26] analyzed the thermodynamic mechanism of thermoacoustic self-excited oscillation. Li Z et al. [27] presented the process of forming thermoacoustic self-excited oscillation in a thermoacoustic engine by means of phase space analysis. Hu X et al. [28] introduced the onset criterion for a thermoacoustic oscillator based on Nyquist instability criterion, which is an important criterion in the theory of electronic oscillator. Qiu L et al. [29] proposed a novel model based on the circuit network analogy to predict the onset temperature of a standing-wave thermoacoustic engine. The mechanism of parameter oscillation has been put forward by Wu J [16], Chen X [17] and Zhang C [18]. They established a network model with three-frequency parameters to research nonlinear problems in thermoacoustic oscillation.

To these authors' knowledge of existing literature, so far there are no time-dependent equations revealing the physical mechanism of parametric excitation of thermoacoustic oscillation. We found great illumination in Taoism, which originated in ancient China. Just as a combination of rigidity and flexibility allows for a system or things to run in order, likewise, the coupling of the inertance with the capacitance in a thermoacoustic network can cause a resonance. The aim of the present research is to reveal theoretically the mechanism of parametric excitation for a thermoacoustic system. Based on former works, a mathematic model with periodic time variant equation will be established in this paper. The nonlinear effect of the thermoacoustic parameter oscillation will be investigated by drawing a parallel between a thermoacoustic network and a nonlinear equation of parametric excitation. A technique called "averaging method" will be employed to solve the equation of nonlinear parametric vibration. The condition of a homeostatic periodic motion is derived and a limit cycle is found. The stability of the limit cycle will be analyzed using the method of Lyapunov function. The results obtained here will provide a method revealing the evolutionary mechanism of a thermoacoustic parameter oscillation system.

2. Equation of the thermoacoustic parameter exciting

The object for the present investigation is a standing-wave thermoacoustic prime mover in our laboratory. The photographic view of the setup is presented in Fig. 1. The thermoacoustic oscillation is inspired and sustained by the thermoacoustic core (including stack, hot and cold heat exchangers) in nature. A schematic diagram of the core is illustrated in Fig. 2. T_h and T_c in the Fig. 2 denote the temperatures of hot and cold heat exchangers, respectively. Assume the gas displacement oscillation along the z direction (along the axis of the thermoacoustic stack). We begin by examining the governing equations of the fluid parcels inside the stack. The basic equations of fluid mechanics are the continuity equation, the momentum equation, the energy equation and the state equation, corresponding to conservation of mass, conservation of momentum, conservation of energy and the thermodynamics relation described by the state of the gas parcel, respectively. They can be given as following [26]

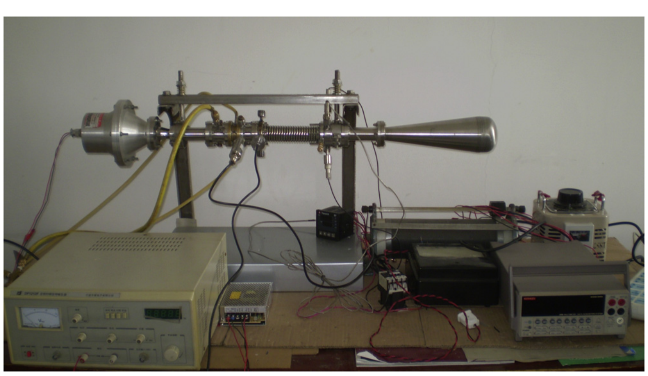

Fig. 1. The photographic view of the standing-wave thermoacoustic prime mover.

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla\cdot(\rho u^{\rightarrow}) = 0 \\ \dfrac{\rho\, \mathrm{d}u^{\rightarrow}}{\mathrm{d}t} = \rho F^{\rightarrow} - \nabla p + \mu \nabla^2 \vec{u} \\ \rho C_p\left(\dfrac{\partial T}{\partial t} + u^{\rightarrow}\cdot\nabla T\right) - \left(\dfrac{\partial p}{\partial t} + u^{\rightarrow}\cdot\nabla p\right) = k\nabla^2 T \\ p = \rho R T \end{cases} \tag{1}$$

where ρ, $\vec{u}$, P, T, μ, C_p, and k are the density, the velocity, the pressure, the temperature, the viscosity, the heat capacity and the thermal conductivity, respectively. The fluid is assumed to be an ideal gas in equation (1).

Under the oscillation, the density, the velocity, the pressure, the temperature can be seen as two parts: a mean term and a fluctuation term. By means of the conventional method time-dependent variables are expressed as

$$Z(\vec{r},t) = Z_m(\vec{r}) + Z'(\vec{r},t) \tag{2}$$

where $Z(\vec{r},t)$ indicates the $\rho(\vec{r},t)$, $u(\vec{r},t)$, $P(\vec{r},t)$ or $T(\vec{r},t)$, $Z'(\vec{r},t)$ is usually the disturbed quantity corresponding to $Z(\vec{r},t)$.

Using equations (1) and (2), the governing equation (1) can been approximately rewritten as

$$\begin{cases} \dfrac{\partial \rho'}{\partial t} + \rho_m \dfrac{\partial u'}{\partial z} = 0 \\ \rho_m\left(\dfrac{\partial u'}{\partial t} + u'\dfrac{\partial u'}{\partial z}\right) = -\dfrac{\partial P'}{\partial z} + \mu\nabla_\perp^2 u' \\ \rho_m C_p\left(\dfrac{\partial T'}{\partial t} + u'\dfrac{dT_m}{dz} + u'\dfrac{\partial T'}{\partial z}\right) - \left(\dfrac{\partial P'}{\partial t} + u'\dfrac{\partial P'}{\partial z}\right) = k\nabla_\perp^2 T' \\ \rho' = \dfrac{\rho_m}{P_m}P' - \dfrac{\rho_m}{T_m}T' \end{cases} \tag{3}$$

where $\nabla_\perp^2$ is the transverse component of the Laplacian, which depends on the geometry of the fluid channel. In order to underline the influence of time on the evolution of the system, we average the values of u' and T' in the cross-sectional area of the fluid channel. Using the variable separation method, we have

$$u' = u_t(t)u_\perp(x,y)v(z) = u_t u_z \tag{4}$$

with $u_z = v(z)/A\int_0^A u_\perp(x,y)\mathrm{d}s$. Here $u_t = u_t(t)$ denotes the time component of the velocity, $v(z)$ denotes the longitudinal component of the velocity, $u_\perp(x,y)$ denotes the transverse component of the velocity and A denotes the cross-sectional area of the fluid channel.

$$T' = T_t(t)T_\perp(x,y)T_L(z) = T_t T_z \tag{5}$$

with $T_z = T_L(z)/A\int_0^A T_\perp(x,y)\mathrm{d}s$. Here $T_t = T_t(t)$ is the time component of the temperature, $T_L(z)$ is the longitudinal component

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