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Optimal design of unequal heat flux elements for optimized heat transfer inside a rectangular duct

M.R. Hajmohammadi [a,*], M. Rahmani [b], A. Campo [c], O. Joneydi Shariatzadeh [d]

[a] Department of Mechanical Engineering, Amirkabir University of Technology, Tehran 158754413, Iran
[b] Department of Mechanical Engineering, Arak University of Technology, Arak 41167-38181, Iran
[c] Department of Mechanical Engineering, The University of Texas at San Antonio, San Antonio, TX 78249, USA
[d] Department of Energy Technology, Lappeenranta University of Technology, P.O. Box 20, 53851 Lappeenranta, Finland

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ABSTRACT

When the total amount of thermal energy is fixed in a device, it is important to keep the highest temperatures (hot spots) of the device at the minimum level. The present paper deals with the optimal design of heat flux elements mounted on the outer walls of a rectangular duct. The total amount of heat load is fixed and is transferred by in-duct laminar forced convection. The objective is to minimize the hot spots temperature under the platform of constructal design. A numerical simulation is carried out to calculate the hot spots temperatures. The numerical results suggest that the equal heat flux elements (uniform heating) customarily used in industry must be avoided. By conducting a detailed optimization process, it is shown that there exists an optimum 'descending' distribution for the unequal heat flux elements that minimizes the hot spot temperatures. The influence of Graetz number and the number of unequal heat flux elements on the temperature reduction is studied. For instance, compared with the case of uniform heating, it is shown that the hot spot temperature is reduced up to 25% in the case of four unequal heat flux element under the influence of intermediate values of Graetz number.

1. Introduction

The study of optimized heat transfer devoted to multi scale flows has become a subject of interest to estimate specific configurations that render maximum heat transfer density [1,2] in the last decades. The main objective is to enrich the flow configuration with some degree of freedom to make easier and greater access to its heat transfer currents [3,4]. This is the essential concept of constructal theory proposed by Bejan [3] that states "for a finite-size system to persist in time, it must evolve in such a way that provides easier access to the imposed currents flowing through it". Using constructal theory, this natural behavior can be conceived as a phenomenon of configuration. Bejan and Lorente [4] reported a comprehensive review on the constructal design of heat transfer systems and later determined the best configuration for freely morphing convective systems under the constraint of fixed volume.

In heating devices, when temperature exceeds an allowable level, the system starts to fail. Therefore, in devices with fixed amount of overall heat load, it is important to keep the highest temperatures (hot spots) of the device at the minimum level. To accomplish this goal, constructal design [3,4] suggests optimizing the geometric or physical variables capable of change. For example, geometric optimization of finned surfaces [5–11], cavities [12–19] and highly conductive pathways [20–28] embedded in electronic heat generating devices has been the subject of interests. Lorenzini et al. [7], for instance, have optimized the geometric parameters associated with a T–Y shape of a finned surface, such that the maximum temperature (hot spot) is minimized. In addition to systems with conduction [5–28], constructal design has also been applied in convective heat transfer devices [29–39]. For example, Bejan and Sciubba [29] optimized the space between arrays of heat sources cooled by forced convection, such that the maximum temperature is minimized. Bejan [30] reported that the same technique can be utilized when cooling occurs with natural convection. Bejan and Fautrelle [31] optimized the number of additional heat sources that were inserted in the flow entrance region between the current heat sources, such that the maximum temperature of the system reaches the minimum level. Chen et al. [32] conducted experiments to investigate the effects of different arrangements of heating elements on the cooling performance of a simulated electronic package. Analytical calculations were carried out by da Silva et al. [33] and Hajmohammadi et al. [34,39] to

* Corresponding author. Tel.: +98 9125486617; fax: +98 2166419736.
E-mail address: mh.hajmohammadi@yahoo.com (M.R. Hajmohammadi).

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determine the optimal insulated spacing between heat sources mounted on a plate [34] or on a round pipe [39] cooled by forced convection. Efficiency increase by a non-uniform heat sink distribution in a solar thermochemical reactor is investigated by Tescari et al. [40]. Geometric optimization has also been developed for fuel cells [41], thermoelectric coolers [42] and micro-channels [43,44].

In this paper, the heat flux elements mounted on the heated outer wall of a rectangular duct are considered unequal (non-uniform heating). It is shown that the equal heat flux elements (uniform heating) that are customarily used in industry must be avoided. By conducting a detailed optimization process, it is shown that there exists an optimum 'descending' distribution for the unequal heat flux elements that minimizes the hot spot temperatures. The reduction in temperature is found dependent on the number of unequal heat flux elements and the Graetz number.

2. Physical system

Shown in Fig. 1, the schematic of the system consists of a rectangular duct of length L with n heat flux elements of length $l = L/n$ attached at the outer surface of the duct. The rectangular face of the duct is with dimensions a and b and a/b is the aspect ratio of the duct. The coordinates, x_1, x_2 and x_3 are orthogonal. Each of the heat flux elements (heaters) generates uniform heat flux, q'', however the amount of heat flux is different for each element, i.e., $q''_1 \neq q''_2 \neq \cdots \neq q''_n$. The amount of heat flux at different x_3 (x_3 represents the coordinate in flow direction) can be expressed as follows:

$$q''(x_3) = q''_1 \hat{q}_i; \quad \hat{q}_i = \begin{cases} 1 \quad 0 \le x_3 < \frac{L}{n}(i=1) \\ \frac{q''_i}{q''_1} \quad (i-1)\frac{L}{n} \le x_3 < i\frac{L}{n}; \left(i = 2:n-1\right) \\ \frac{q''_n}{q''_1} \quad L - \frac{L}{n} \le x_3 \le L; \left(i=n\right) \end{cases} \tag{1}$$

where $\hat{q}_i$ represents the ratio of the heat generation rate of the i-th heater to that of the first heater. Although the heat flux elements are unequal, $q''_1 \neq q''_2 \neq \cdots \neq q''_n$, the total heat removal rate from the duct is fixed, i.e.

$$\frac{\sum_{i=1}^{n} q''_i}{n} = q''_{\text{ave}} = \text{Const.} \tag{2}$$

Here, q''_{ave} is the mean value of heat generation rates, q''_i. In this paper, fixed duct dimensions together with the constraint of fixed flow rate (or pressure drop) are assumed. The fluid flow is assumed to be laminar and incompressible. Velocity develops from uniform inlet velocity, U_i and temperature develops from uniform inlet

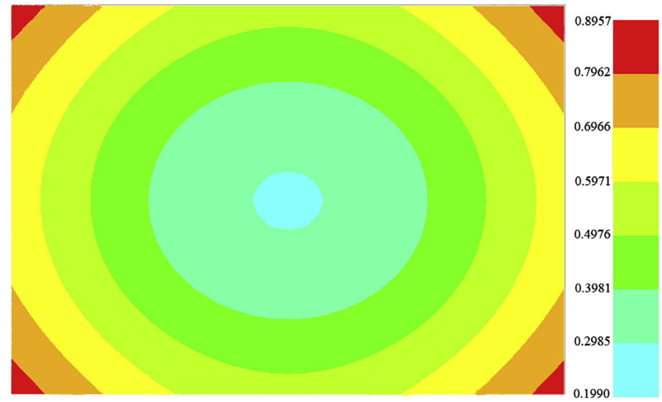

Fig. 2. The temperature field (x_1, x_2) at the end of the duct with 1.43 aspect ratio when $\hat{q}_2 = 0.61$, $n = 2$ and $Gz^{-1} = 0.1$.

temperature, T_i. The duct thickness is assumed thin to neglect the effects of wall conduction. The following trio set of dimensionless variables are defined as

$$\left(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{l}\right) = \frac{(x_1, x_2, x_3, l)}{L}; \quad (\hat{u}_1, \hat{u}_2, \hat{u}_3) = \frac{(u_1, u_2, u_3)}{U_\text{i}}; \quad \hat{p} = \frac{P}{\rho U_\text{i}^2} \tag{3}$$

where x_1, x_2 and x_3 are the coordinates illustrated in Fig. 1. u_1, u_2 and u_3 are the x_1-component, x_2-component and x_3-component of the velocity, P is the pressure and ρ is the fluid density. The dimensionless temperature is defined as

$$\hat{T}(\hat{x}_1, \hat{x}_2, \hat{x}_3) = \frac{T(\hat{x}_1, \hat{x}_2, \hat{x}_3) - T_\text{i}}{T_\text{L} - T_\text{i}} \tag{4}$$

where T_L stands for the peak temperature when the duct is heated with uniform/constant heat flux, q''_{ave}; and T_i is the inlet fluid temperature. Further, the Graetz number is defined as

$$Gz = \frac{Re\,Pr}{L/D_\text{e}} \tag{5}$$

or its equivalent

$$Gz^{-1} = \frac{L/D_\text{e}}{Re\,Pr} \tag{6}$$

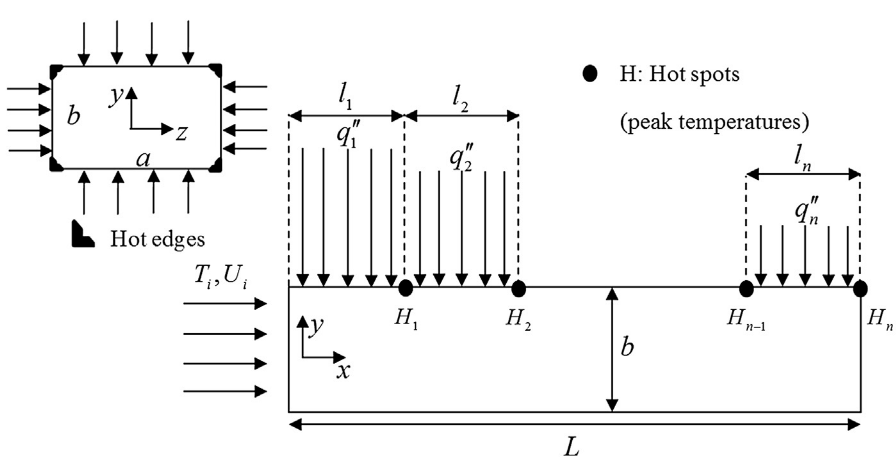

Fig. 1. Geometric and coordinate system definition of the physical system.