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Heat transfer and entropy generation in the parallel plate flow of a power-law fluid with asymmetric convective cooling

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ABSTRACT

The heat transfer and entropy generation in the parallel plate flow of a power-law fluid are analyzed. Asymmetric convective cooling is included in the analysis by considering thermal boundary conditions of the third kind. Using the known velocity profile, the temperature field is analytically derived. Conditions for minimum entropy generation are determined.

1. Introduction

Due to their widespread presence in many practical situations, heat transfer problems in fluid systems have attracted the attention of researchers for a long time. While the original focus was on Newtonian fluids (a classical reference is the work by Shah and London [1]), more recently interest in non-Newtonian fluids has increased [2–8]. This has been provoked by the importance for many industries (including polymer processing and the food industry) of thermal conditions in the flow of such systems through pipes, ducts and devices of different shapes. For instance, when dealing with polymeric materials, a key factor for the quality of the final product is a proper temperature control. Since the actual operating conditions are in general very complex, the analysis of relatively simple but tractable problems is usually taken as a useful resource to gain some insight. So it is not surprising that many such developments have appeared in the literature, even quite recently, which include fluid flow and heat transfer in cylindrical conduits or between parallel plates under different thermal boundary conditions [9–18]. Another tool which has become rather popular in recent times (see for instance Refs. [14–19]) for the analysis of these problems is the use of the second law of thermodynamics, in particular in what concerns the generation of entropy within the system. This generation is caused by the various irreversibilities present in the process under investigation and so a detailed knowledge of the parameters determining such irreversibilities may prove crucial for specifying the best operating conditions. In fact, as Bejan has pointed out [20], good engineering heat transfer design in problems where either heat transfer augmentation or thermal insulation are required usually involves the minimization of entropy generation. Interestingly enough the same approach has been recently used in different contexts by several authors, see for instance Ref. [21].

A few years ago, using the Entropy Generation Minimization method [20,22] two of us [23] showed that the entropy generation in the viscous flow between parallel plates with asymmetric convective cooling displayed a minimum for given values of the ambient temperature and the upper and lower plate Biot numbers. The same effect has also been found for other problems [24] that involve the flow of Newtonian fluids. The question then naturally arises as to whether the consideration of a non-Newtonian fluid will affect, and if so to what extent, the features that stem out of previous investigations. In fact, the analysis [25] of the heat transfer problem in the zero-mean oscillatory flow of a Maxwell fluid flowing between parallel plates with convective cooling suggests

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that the effects of viscoelasticity may produce heat transfer enhancement with respect to that of a Newtonian fluid under similar operating conditions. To further address the question posed above, in this paper we have examined the problem of heat transfer and entropy generation in the fully developed parallel plate flow of a power-law fluid with asymmetric convective cooling. The choice of the power-law fluid to carry out this analysis is due to two reasons. On the one hand, it includes the Newtonian fluid as a special case. On the other hand, although it has also been used to analyze heat transfer and entropy generation in different kinds of flow [26–47], the parallel plate flow under asymmetric convective cooling has not been examined to our knowledge so far.

The paper is organized as follows. In the next Section, we provide a brief description of the power-law fluid and of the assumptions under which our problem will be set, including all the governing equations. This is followed in Sec. 3 by the explicit determination of the velocity and temperature fields and hence of the corresponding local and global entropy generation and their analysis for both pseudoplastic and dilatant fluids The paper is closed in Sec. 4 with some further discussion and concluding remarks.

2. The model fluid and the governing equations

In this Section we will start by stating the problem under consideration. This involves writing down the equations that determine the velocity and temperature fields in the fully developed parallel plate flow of a power-law fluid with asymmetric convective cooling. A power-law fluid is a type of generalized Newtonian fluid for which the shear stress τ is given by Ref. [48]

$$\tau \equiv -m\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}, \tag{1}$$

where y is the transversal coordinate, u the axial fluid velocity, m is the flow consistency index, $\partial u/\partial y$ is the shear rate or the velocity gradient perpendicular to the plane of shear and n is the flow behavior index. If $n < 1$ the fluid is pseudoplastic (*e.g.* styling gel) while if $n > 1$ it is dilatant (rarely encountered, *e.g.* an uncooked paste of cornstarch and water). If $n = 1$, then it is the Newtonian fluid. In Fig. 1 we show a schematic diagram of the system we want to examine.

For the sake of deriving analytical results, we will take the following simplifying assumptions. We consider a steady laminar flow of an incompressible power-law fluid that takes place between parallel rigid plates separated by a distance $b = 2a$. The flow is driven by a constant pressure gradient, $\partial p/\partial x$, in the axial x-direction. We assume that the parallel plates are infinite so that border effects are neglected and the velocity and temperature profiles are fully developed. For the solution of the momentum balance equation we assume that the velocity satisfies the no slip condition at the plates. In turn, the heat transfer equation is solved using boundary conditions of the third kind that indicate that the normal temperature gradient at any point in the boundary is assumed to be proportional to the difference between the temperature at the surface and the external ambient temperature T_a, which is assumed constant. With these conditions the amount of heat entering or leaving the system depends on the external temperature as well as on the convective heat transfer coefficient. A fundamental assumption in this problem is that the heat transfer coefficient of each plate is different and therefore, we have an asymmetric convective cooling. We also assume that natural convection is absent and that the thermal conductivity of the fluid, k, is constant.

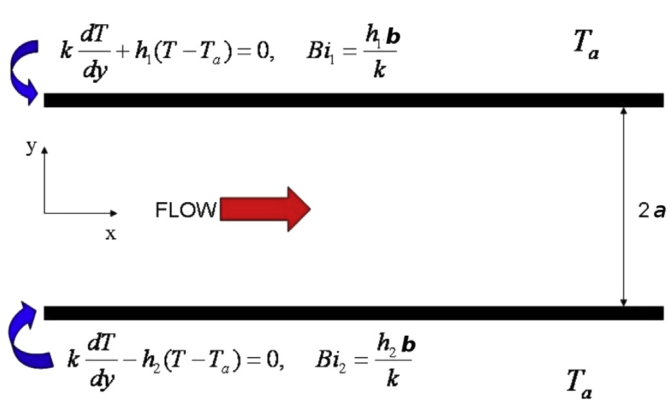

Fig. 1. Schematic representation of the problem under analysis.

Given the previous assumptions, let us now express the balance equations for momentum and energy along with their boundary conditions. In dimensionless terms, upon substitution of the expression for the shear stress as given in Eq. (1), the momentum balance equation turns out to be the following

$$\frac{\mathrm{d}}{\mathrm{d}y^*}\left(\left|\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right|^{n-1}\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right) = 1, \tag{2}$$

where the dimensionless velocity u^* is normalized by the maximum axial fluid velocity $u_0 = ((1/m)(\partial p/\partial x))^{1/n}b^{n+1/n}$, while the dimensionless transversal coordinate is given by $y^* = y/b$. The solution of Eq. (2) must satisfy the no slip boundary conditions

$$u^*(1/2) = u^*(-1/2) = 0. \tag{3}$$

In turn, the energy balance equation results

$$\frac{\mathrm{d}^2T^*}{\mathrm{d}y^{*2}} + \left|\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right|^{n-1}\left(\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right)^2 = 0. \tag{4}$$

where the dimensionless temperature is defined as $T^* = k(T-T_a)b^{n-1}/mu_0^{n+1}$.

According to our assumptions, we now consider boundary conditions of the third kind for the thermal problem. As mentioned earlier, this has to do with the fact that the amount of heat going into or out of the system depends on the external temperature as well as on the (effective) convective heat transfer coefficients. The latter include both the thermal resistance of the plates and the external convective heat transfer coefficients. Therefore, the boundary conditions associated to our heat transfer problem [*c.f.* Eq. (4)] are given by

$$\frac{\mathrm{d}T^*}{\mathrm{d}y^*} + \mathrm{Bi}_1T^* = 0,\ \text{at}\ y^* = 1/2 \tag{5}$$

and

$$\frac{\mathrm{d}T^*}{\mathrm{d}y^*} - \mathrm{Bi}_2T^* = 0,\ \text{at}\ y^* = -1/2. \tag{6}$$

In Eqs. (5) and (6) the Biot numbers $\mathrm{Bi}_1 = (h_{\mathrm{eff}})_1b/k$ and $\mathrm{Bi}_2 = (h_{\mathrm{eff}})_2b/k$ are the dimensionless expressions of the effective convective heat transfer coefficients of the upper and lower plates, $(h_{\mathrm{eff}})_1$ and $(h_{\mathrm{eff}})_2$, respectively, which, due to our former assumptions, turn out to be different and k is the heat conductivity of the fluid. The effective heat transfer coefficients are defined as