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Optimal connecting elements allocation in linear consecutively-connected systems with phased mission and common cause failures

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ABSTRACT

This paper considers linear consecutively-connected systems (LCCSs) subject to multiple phases of mission and common cause failures. Many real-world systems such as communication networks and flow transmission systems can be modeled as a phased-mission LCCS (PM-LCCS) that consists of linearly ordered nodes with some of them containing one or multiple connection elements (CEs). Each of these CEs provides a connection between its host node and a certain number of downstream nodes depending on the connection range of the CE; they work together to provide path connectivity between a pair of source and destination nodes specified in the transmission task. Common cause failures (CCFs) can occur in a node and destroy all CEs located in that node. The system fails if the source and destination nodes are disconnected. The considered PM-LCCS must perform a sequence of transmission tasks over multiple non-overlapping phases that can be subjected to different stresses and environment conditions, causing dynamics in elements failure behavior. During each phase the system may be required to provide continuous connection along a different path of nodes, and common nodes may appear in different paths causing statistical dependence across the phases. In this paper, the problem of optimal allocation of CEs to nodes in PM-LCCSs with CCFs is formulated and solved for maximizing the overall mission reliability. The proposed methodology includes a recursive reliability evaluation algorithm for PM-LCCSs which takes into account CCFs, phase dependence as well as dynamics in path configuration and elements failure behaviors. A genetic algorithm is then adapted for solving the formulated optimal allocation problem for PM-LCCSs with CCFs. The proposed approach is illustrated using examples.

1. Introduction

A linear consecutively-connected system (LCCS) consists of M linearly ordered nodes. Some nodes contain statistically independent connecting elements (CEs) aiming to accomplish a particular type of transmission or connection tasks between a specified pair of source and destination nodes. Each CE assumes binary states. In its working state, CE i can provide a connection between its host node and $L(i)$ next nodes; in its failed state, the CE cannot provide any connection. The system fails if its source and destination nodes are disconnected. A classic example of LCCSs is flow (e.g., gas, oil) transmission pipe lines with pressure generating units, particularly, pumps being CEs [1]. Other examples of LCCSs include radio relay systems with re-transmitters, and wireless sensor networks [2–4].

The concept of LCCSs was introduced as a generalization of binary linear consecutive-k-out-of-n: F systems [1,5] and consecutively-connected systems with binary-state elements [6,7]. Reliability modeling and analysis of LCCSs and their multi-state generalization were performed in [4,8–11]. When an LCCS consists of non-identical CEs (with different failure probabilities and connection ranges), its reliability can be greatly affected by the allocation of CEs among the system nodes. Thus, the optimal CE allocation problem arises. Such problem has been formulated and solved by Malinowski and Preuss [12] and Levitin [13]. In [14] this optimization problem was solved for LCCSs subject to common cause failures that can occur in each node containing several CEs.

The optimal CE allocation problem in LCCSs has been solved mostly for LCCSs that do not change their task and path configuration during the considered mission time. However, in many situations a system has to perform a different task during different phases of its mission [15–18]. During each phase, the system may be subject to different requirements and different levels of stresses and environmental conditions. Thus, the system configuration and

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Nomenclature

H	number of phases in the system mission
M	total number of nodes
N	number of available CEs
D_h	connection path in phase h
$\|D_h\|$	number of nodes in D_h
$d_h(j)$	j-th node in D_h
$\boldsymbol{\Theta}$	set of CEs $\Theta=\{e_1,\ldots,e_N\}$
$P(\boldsymbol{\Theta})$	power set of $\boldsymbol{\Theta}$
$\omega(m)$	set of CEs located at node m
Ω	CE allocation in LCCS $\Omega=\{\omega(1), \omega(2),\ldots, \omega(M)\}$
S	integer string representing the CE allocation in the genetic algorithm
$s(j)$	node where CE j is located
$\varepsilon_m(h)$	probability of CCF of node m in phase h
$R(\Omega)$	reliability of PM-LCCS for the given CE allocation Ω
$L(i)$	connection range of CE i in working state
τ_h	duration of phase h
$x_h(i)$	state of binary CE i in the end of phase h
$\boldsymbol{X}_h$	vector of numbers of CEs failed before the end of phase h
$\phi_h(\boldsymbol{X}_h)$	system state acceptability function in phase h
F_i	baseline failure distribution of CE i
$\beta(i),\eta(i)$	shape, scale parameters of F_i in the case of Weibull distribution
$F_{ij}(h, t)$	stress dependent failure time distribution of CE i located at node j
$p_i(h)$	conditional reliability of CE i at phase h
$q_i(h)$	conditional unreliability of CE i at phase h
$\Phi_{ij}(h)$	cumulative failure probability of CE i located at node j at the end of phase h
$f_{ij}(h)$	probability that CE i located at node j fails in phase h
$\alpha_j(h)$	acceleration factor during phase h at node j
$0(\boldsymbol{Y})$	number of zeros in binary vector $\boldsymbol{Y}$
$Q_h(\boldsymbol{Y},\sigma)$	occurrence probability of the combination of CE failures that transits the set of failed CEs from $\boldsymbol{Y}-\boldsymbol{Y}_\sigma$ to $\boldsymbol{Y}$ in phase h
$Z_{h,\boldsymbol{Y}}$	probability of system state $\boldsymbol{Y}$ in phase h
$\lfloor x \rfloor$	floor operation that returns the maximal integer number that does not exceed x
$\mathrm{mod}_2 x$	modulo-2 function of integer argument x that returns 1 when x is odd, and 0 if x is even; $\mathrm{mod}_2x=x-2\lfloor x/2\rfloor$
Λ	solution fitness in the genetic algorithm
t_k	the most remote node that all the elements located at nodes $d_h(1)$, $d_h(2)$, …, $d_h(k)$ can reach if the path D_h is endless
Δ	set of CE numbers {1,…,N}

Acronym

CE	connection element
CCF	common cause failure
CEM	cumulative exposure model
GA	genetic algorithm
LCCS	linear consecutively-connected system
PM-LCCS	phased-mission LCCS

element failure behavior may vary from phase to phase. In addition, the statistical dependence across phases for elements appearing in multiple phases further complicates the analysis. Examples of such phased-mission LCCSs (PM-LCCSs) are flow transportation and information networks, which are highly branched and consecutively perform different transmission tasks between different pairs of source and destination nodes during different phases of their functioning (see Fig. 1 for a specific example of PM-LCCSs).

Modeling and optimization of PM-LCCSs must consider both the aforementioned dynamic behaviors of system and elements as well as the statistical phase-dependence, which poses unique challenges to existing methods for LCCSs. Recently, Levitin et al. have, for the first time, formulated and solved the optimal allocation problem in PM-LCCSs to address those challenges [3]. However, they do not address effects of common-cause failures (for example, bad weather conditions, malicious attacks or other location dependent stresses) which can occur to a node within the PM-LCCS causing immediate failures of all CEs located in this node.

In this paper, we extend the works [3] and [14] by considering common-cause failures (CCFs) in the reliability evaluation and optimization of PM-LCCSs. The proposed methodology allows the

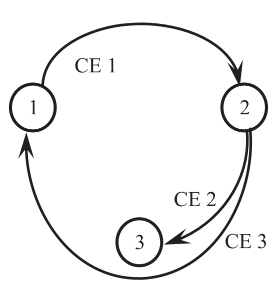

Fig. 1. Two-phase LCCS with $M=N=3$.

probabilities of CCFs to be dependent on the node and phase in which they occur. In addition, different from [3] where at most one CE can be allocated to a node, the proposed method is more general allowing more than one CE to be allocated at the same node. The benefit of such general CE allocation as well as the effects of CCFs on the analysis and optimization results are illustrated by detailed examples.

The remainder of the paper is organized as follows. Section 2 describes the system and gives the formulation of the considered optimal CE allocation problem. Section 3 presents assumptions used in this work. Section 4 describes the reliability evaluation algorithm for PM-LCCS subject to CCFs. Section 5 gives an illustrative example of the proposed reliability evaluation algorithm. Section 6 presents the optimization technique used in this work. Section 7 illustrates the proposed methodology using a practical example of wireless sensor networks. Section 8 presents conclusions.

2. The model of PM-LCCS

The system consists of M statistically independent nodes used in H consecutive, non-overlapping phases of mission. The list of nodes used in the transmission process changes with the phases. Thus, some nodes can belong to all phases, and some only to specific phases. In each phase h the system has to provide connectivity along a specified path D_h, which consists of a given sequence of $|D_h|\le M$ different numbers of nodes: $D_h=\{d_h(1), \ldots, d_h(|D_h|)\}$ $(1\le d_h(j)\le M$ for any $1\le j\le |D_h|)$.

There are N different CEs which are to be distributed among the nodes in the PM-LCCS. For any specific CE allocation one can define a set of CEs $\omega(m)$ located at each node m. Any CE i when it is functioning can provide a connection between its host node and $L(i)$ next nodes in the path. Define $x_h(i)$ as the Boolean state