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Inverse Gaussian process models for degradation analysis: A Bayesian perspective

Weiwen Peng [a], Yan-Feng Li [a], Yuan-Jian Yang [a], Hong-Zhong Huang [a,*], Ming J. Zuo [a,b]

[a] School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, Chengdu 611731, Sichuan, China
[b] Department of Mechanical Engineering, University of Alberta, Edmonton, AB, Canada T6G 2G8

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ABSTRACT

This paper conducts a Bayesian analysis of inverse Gaussian process models for degradation modeling and inference. Novel features of the Bayesian analysis are the natural manners for incorporating subjective information, pooling of random effects information among product population, and a straightforward way of coping with evolving data sets for on-line prediction. A general Bayesian framework is proposed for degradation analysis with inverse Gaussian process models. A simple inverse Gaussian process model and three inverse Gaussian process models with random effects are investigated using Bayesian method. In addition, a comprehensive sensitivity analysis of prior distributions and sample sizes is carried out through simulation. Finally, a classic example is presented to demonstrate the applicability of the Bayesian method for degradation analysis with the inverse Gaussian process models.

1. Introduction

Modern products evolve from generation to generation. It naturally gives rise to the continuing cutting down of time-to-market and the ever-increasing pace of new products appearing on the market. Meanwhile, product reliability has become an indispensable aspect of customer expectation. Increased reliability expectation with lower cost has become a critical issue for companies to deliver competitive products. Methods such as condition monitoring and degradation analysis are developed for reliability analysis of modern products. Degradation analysis is demonstrated as a significant toolkit, especially for the ones that are subjected to limited test time and sample size [1]. A comprehensive guide to degradation analysis is previously introduced by Meeker and Escobar [2]. Followed by a great amount of published papers, degradation related methods are introduced for various fields of reliability, which include reliability tests [3–5], reliability analysis [6–9], and fault prognostics [10–13]. Degradation modeling and parameter estimation are two indispensable aspects for the implementation of degradation analysis. A suitable degradation model is the key point for degradation characterization and reliability representation of a product. Meanwhile, a flexible estimation method is the key point for reliability assessment and degradation inference of a product. A high-precision reliability analysis of modern products consequently relies heavily on these two critical aspects of degradation analysis.

Considering the research on degradation modeling, the stochastic process based models are generally used [7,9]. Two most common classes of stochastic process are the gamma and the Wiener processes. These two classes have been well studied in the literature. The gamma process and its extensions in degradation modeling have been investigated in the works [14–16]. The applications of the Wiener process and its extensions in degradation modeling have also been investigated in the works [17,18]. Recently, the inverse Gaussian (IG) process has been reported as an attractive and flexible model for degradation modeling by Wang and Xu [19]. It has been demonstrated by them that the IG process model is more suitable than the Wiener and the gamma processes models for degradation modeling in some applications. Qin et al. [20] has also demonstrated the flexibility of IG process for degradation modeling through the application to the reliability analysis of energy pipelines. These two works were based on a simple IG process model. Ye and Chen [21] investigated the physical interpretation of IG process for degradation modeling and further introduced three IG process models with random effects by extending the simple IG model. An inverse normal-gamma mixture of an IG process model was also proposed by Peng [22]. These models were useful for the situations that random effects were associated with the degradation mean and variance of products. However, considering the situations that the random effects affect solely on the degradation mean, these models were limited due to the correlation between degradation mean and variance through random effects parameter. A classic example is the degradation data of

* Corresponding author. Tel.: +86 28 6183 0248; fax: +86 28 6183 0229.
E-mail address: hzhuang@uestc.edu.cn (H.-Z. Huang).

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Nomenclature

IG inverse Gaussian
MLE maximum likelihood estimation
EM expectation maximization
PDF probability density function
CDF cumulative distribution function
MCMC Markov chain Monte Carlo method
RD random drift IG process
RV random volatility IG process
RDV random drift-volatility IG process
$\boldsymbol{\theta}$ parameters of an IG process model
$\boldsymbol{\theta}^F$ parameters without random effects (fixed parameters)
$\boldsymbol{\theta}^R$ parameters with random effects (random parameters)
$\boldsymbol{\theta}^H$ hyper-parameters of probability distributions for the random parameters
$\pi(\boldsymbol{\theta})$ prior distribution
Y_i degradation data of the ith product with $i=1,\ldots,n$
$\boldsymbol{\theta}_i^R$ random parameters of the ith degradation process with $i=1,\ldots,n$
$\pi(\boldsymbol{\theta}_i^R|\boldsymbol{\theta}^H)$ prior distribution of random parameters $\boldsymbol{\theta}_i^R$ with hyper-parameters $\boldsymbol{\theta}^H$
Δy_{ij} degradation increment
$L(Y|\boldsymbol{\theta})$ likelihood function
$f(\Delta y_{ij}|\boldsymbol{\theta}^F)$ PDF of degradation increment under an IG process model without random effects
$f(\Delta y_{ij}|\boldsymbol{\theta}^F,\boldsymbol{\theta}_i^R)$ PDF of degradation increment under an IG process model with random effects
$p(\boldsymbol{\theta}^F,\boldsymbol{\theta}^R,\boldsymbol{\theta}^H|Y)$ posterior distribution
$Y_S(t)$ degradation process with a simple IG process model
$\Lambda(t)$ monotone increasing function in an IG process model
θ_Λ parameters of function $\Lambda(t)$
$\Phi(\cdot)$ CDF of a standard normal distribution
$\phi(\cdot)$ PDF of a standard normal distribution
$Pr(\cdot)$ probability of an event
$f(y|a,b)$ PDF of an IG distribution
$F(y|a,b)$ CDF of an IG distribution
$\mathrm{IG}(\mu\Delta\Lambda,\lambda\Delta\Lambda^2)$ IG process with function $\Lambda(t)$ and parameter μ and λ
$f_S(y|\mu\Lambda(t),\lambda\Lambda^2(t))$ PDF of a simple IG process model
$R_S(t|\mu\Lambda(t),\lambda\Lambda^2(t))$ reliability function of a product with a simple IG process model
$\mathrm{TN}(\omega,\kappa^{-2})$ truncated normal distribution with mean ω and variance κ^{-2}
$\mathrm{Uniform}(a,b)$ uniform distribution with boundary $[a,b]$
$\mathrm{Gamma}(\delta,\gamma)$ gamma distribution with shape parameter δ and rate parameter γ
Y_S degradation data with a simple IG process model
$L_S(Y_S|\mu,\lambda,\theta_\Lambda)$ likelihood function of Y_S with a simple IG process model
$p(\mu,\lambda,\theta_\Lambda|Y_S)$ posterior distribution of model parameters for a simple IG process model
$R_S(t|Y_S)$ inference of reliability for a product with a simple IG process model
$f_{S,m+1}(y|Y_S)$ prediction of degradation at t_{m+1} for a product with a simple IG process model
$Y_{RD}(t)$ degradation process with a RD model
$f_{RD}(y|\omega,\kappa,\Lambda(t),\lambda)$ PDF of a RD model
$R_{RD}(t|\omega,\kappa,\Lambda(t),\lambda)$ reliability function of a product with a RD model
Y_{RD} degradation data with a RD model
$\boldsymbol{\mu}$ set of random parameters
$l_{RD}(Y_{RD,i},\mu_i|\omega,\kappa,\theta_\Lambda,\lambda)$ likelihood contribution of the ith degradation path $Y_{RD,i}$ with a RD model
$L_{RD}(Y_{RD},\boldsymbol{\mu}|\omega,\kappa,\theta_\Lambda,\lambda)$ likelihood function of Y_{RD} and random parameters with a RD model
$p(\omega,\kappa,\theta_\Lambda,\lambda,\boldsymbol{\mu}|Y_{RD})$ posterior distribution of model parameters for a RD model
$R_{RD}(t|Y_{RD})$ inference of reliability for the product population with a RD model
$f_{RDi,m+1}(y|Y_{RD})$ prediction of degradation at $t_{i,m+1}$ for the ith product with a RD model
$Y_{RV}(t)$ degradation process with a RV model
$\Gamma(\bullet)$ gamma function
$f_{RV}(y|\mu,\Lambda(t),\delta,\gamma)$ PDF of a RV model
$R_{RV}(t|\mu,\Lambda(t),\delta,\gamma)$ reliability function of a product with a RV model
Y_{RV} degradation data with a RV model
$\boldsymbol{\lambda}$ set of random parameters
$L_{RV}(Y_{RV},\boldsymbol{\lambda}|\mu,\theta_\Lambda,\delta,\gamma)$ likelihood function of Y_{RV} and random parameters with a RV model
$p(\mu,\theta_\Lambda,\delta,\gamma,\boldsymbol{\lambda}|Y_{RV})$ posterior distribution of model parameters for a RV model
$R_{RV}(t|Y_{RV})$ inference of reliability for the product population with a RV model
$f_{RVi,m+1}(y|Y_{RV})$ prediction of degradation at $t_{i,m+1}$ for the ith product with a RV model
$Y_{RDV}(t)$ degradation process with a RDV model
$f_{RDV}(y|\omega,\kappa,\Lambda(t),\lambda)$ PDF of a RDV model
$R_{RDV}(t|\omega,\kappa,\Lambda(t),\lambda)$ reliability function of a product with a RDV model
Y_{RDV} degradation data with a RDV model
$L_{RDV}(Y_{RDV},\boldsymbol{\mu}|\omega,\kappa,\theta_\Lambda,\lambda)$ likelihood function of Y_{RDV} and random parameters with a RDV model
$p(\omega,\kappa,\theta_\Lambda,\lambda,\boldsymbol{\mu}|Y_{RDV})$ posterior distribution of model parameters for a RDV model
Y_A degradation data with subscript A representing a specific IG process model
$F_A(y|\theta_A)$ CDF of an IG process model
$S(\tilde{\theta}_A)$ Bayesian χ^2 test statistic
B_p Bayesian χ^2 test probability

the GaAs Laser investigated by Wang and Xu [19] and Ye and Chen [21]. Unit-specific heterogeneity of degradation rate among product population is significant, yet the variance of degradation increments within a specific unit is small. A degradation model with random effects affect solely on the degradation mean is needed for the degradation modeling of this GaAs Laser degradation data. Accordingly, the IG process for degradation modeling still deserves further investigation to make it more versatile for various situations of degradation data.

Considering the research on parameter estimation, the maximum likelihood estimation (MLE) is often the tool of choice to implement parameter estimation for the IG process models. Wang and Xu [19], Ye and Chen [21] and Peng [22] have introduced the MLE for the IG process model using expectation maximization (EM) and bootstrap methods. Nowadays, two typical situations are generally encountered in degradation analysis of modern product, i.e. (1) the degradation analysis with sparse/fragmented degradation observations, and (2) the degradation analysis with