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Reliability evaluation of non-reparable three-state systems using Markov model and its comparison with the UGF and the recursive methods

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ABSTRACT

In multi-state systems (MSS) reliability problems, it is assumed that the components of each subsystem have different performance rates with certain probabilities. This leads into extensive computational efforts involved in using the commonly employed universal generation function (UGF) and the recursive algorithm to obtain reliability of systems consisting of a large number of components. This research deals with evaluating non-repairable three-state systems reliability and proposes a novel method based on a Markov process for which an appropriate state definition is provided. It is shown that solving the derived differential equations significantly reduces the computational time compared to the UGF and the recursive algorithm.

1. Introduction

In the classical system reliability viewpoint, systems are binary-state, in which the components have two states of “working perfect” or “completely failed. “Whereas according to newer approaches, a typical component of a system may work at any performance rate between 0% and 100%, each with a certain probability. These systems that are called multi-state (MSS) have been studied in depth by many researchers such as Lisnianski and Levitin [12] and Tian et al. [17]. While the binary-state system reliability can be obtained through the basic mathematical and statistical relations, most of the research works in MSS reliability problem focus on optimizing the level of a unique system redundancy [12].

System reliability determination of MSSs is extremely hard using mathematical relations (if not possible), because the system states increase rapidly and computational complexity gets high. As an alternative, Ushakov [18] proposed the universal generation function (UGF) approach for the first time. UGF is known as an appropriate method for calculating the reliability and availability of multi-state systems. This method incredibly decreases the number of system state evaluations and makes the system reliability and availability computations easier [1]. Later, Gnedenko and Ushakov [2], Ushakov [19], and Lisnianski and Levitin [12] introduced more applications of the UGF method. Lisnianski et al. [13] utilized UGF to evaluate the reliability of a MSS containing serial, parallel, and series–parallel sub-systems. Ding and Lisnianski [1] showed that the output probability distribution for the entire MSS could be determined by UGF. Moreover, Levitin et al. [5], Levitin and Lisniaski [6,7], and Lisnianski et al. [11] calculated the distribution function of MSSs with series, parallel, and bridge structures by combination of different operators. Further, Levitin et al. [5] presented a redundancy optimization algorithm for a MSS with series–parallel structure using UGF. Kuo and Wan [4] discussed the optimal reliability design in which UGF was employed as the main method in appraising multi-state systems reliability evaluation. Besides, UGF was applied in redundancy allocation problems (RAP) and k-out-of-n systems many times (see for example Tian et al. [17], Lisnianski and Levitin, [12], Ouzineb et al. [16]).

Although UGF is known as a convenient method in order to calculate MSS reliability, because it can evaluate reliability of multi-state systems with series, parallel, series–parallel, and bridge structures ([5–7,11]), but when the number of components in the system increases the required CPU time dramatically increases. This is the main pitfall for which some solutions have

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been proposed in the literature. Wu and Chen [20] offered a recursive algorithm in order to evaluate the reliability of a binary weighted k-out-of-n system. Higashiyama [3] proposed a procedure in order to evaluate the system reliability of a binary weighted k-out-of-n system in fewer time in comparison with the proposed method by Wu and Chen [20]. However, the time and the space complexities of these two methods are similar to the ones of a recursive algorithm proposed by Li and Zuo [10], which is another useful method to evaluate system availability of binary weighted k-out-of-n:G as well as multi-state weighted k-out-of-n: G systems with less required CPU time. Nevertheless, this method cannot be used when the number of components gets large.

In this paper, a novel and effective approach is aimed to calculate system reliability of non-repairable three-state systems by first defining an appropriate system state. Then, using a Markov process and utilizing the Chapman–Kolmogorov theorem [15], differential equations are obtained. We show that solving these differential equations, which leads into state probabilities and consequently the system reliability, requires much less CPU time and provides solutions with the same quality compared to the ones obtained by either UGF or the recursive algorithm. To do this, a brief background on the UGF method is first presented in Section 2. Then, the recursive algorithm is described briefly in Section 3. Next, the problem is defined, the proposed Markov process to model the problem is introduced, and the differential equations are derived in Section 4. Section 5 concerns with evaluating and comparing the performances of the proposed method. Conclusion and recommendations for future research come in Section 6.

2. The universal generation function (UGF)

In order to describe the UGF method, consider a system having n components, where components j; $j=1,2,...,n$, may have k_j different states with certain probabilities of performance rates denoted by an ordering set $g_j=\{g_{j1},g_{j2},...,g_{jk_j}\}$ in which g_{ji_j} represents the performance rate of component j in state $i_j \in \{1,2,...,k_j\}$. The performance rate $G_j(t)$ of component j at time $t \ge 0$ is a random variable taking values in $g_j: G_j(t) \in g_j$. Moreover, let the probabilities associated with different states of component j be the set $P_j=\{p_{j1},p_{j2},...,p_{jk_j}\}$ [12]. Furthermore, $g_{ji_j} \to P_j$ is often called the probability mass function (pmf) [9,1].

As soon as the performance rates of the components are given, the performance rate of a MSS can be determined. Let the system have K different states and g_i be the performance rate of the system in state i; $i=1,2,...,K$. Then, the system performance rate at time $t \ge 0$ will be either a random variable or a random vector that takes values in $\{g_1,...g_i,...,g_K\}$. Thus, the space representing all possible combinations of performance rates for all components is $L^n=\{g_{11},...,g_{1k_1}\}\times\cdots\times\{g_{j1},...,g_{jk_j}\}\times\cdots\times\{g_{n1},...,g_{nk_n}\}$ and the space for all possible values of the entire system performance rates is $M=\{g_1,...,g_K\}$. The transformation $\phi(G_1(t)...G_n(t)): L^n \to M$ that maps the space of performance rates of system components into the space of system performance rates is called the system structure function [12]. Moreover, the total number of possible states (performance rates) of a MSS is

$$K=\prod_{j=1}^{n} k_j \tag{1}$$

Besides, the probability associated with the state i of the system can be obtained as

$$P_i=\prod_{j=1}^{n} p_{ji_j} \tag{2}$$

Denoting the MSS performance rate for state i as

$$g_i=\phi(g_{1i_1},g_{2i_2},...,g_{ni_n}) \tag{3}$$

The probability distribution of the whole system for K combinations of $i_1,i_2,...,i_n$ is

$$g_i=\phi(g_{1i_1},g_{2i_2},...,g_{ni_n}); \quad P_i=\prod_{j=1}^{n} p_{ji_j} \tag{4}$$

where $1 \le i_j \le k_j$, $(1 \le j \le n)$.

The z-transform of a random variable $G_j(t)$ represents its pmf with $p_j=\{p_{j1},p_{j2},...,p_{jk_j}\}$ associated with $g_j=\{g_{j1},g_{j2},...,g_{jk_j}\}$ [12,9]. Eq. (5) shows the probability distribution of the component j, called also individual UGF.

$$u(z)=\sum_{i=1}^{k_j} p_{ji_j} z^{g_{ji_j}} \tag{5}$$

To derive the probability distribution of the entire MSS with an arbitrary structure function ϕ, a general composition operator Ω_ϕ is employed on individual UGF of n components as [12]:

$$\begin{aligned} U(z)&=\Omega_\phi\{u_1(z),...,u_n(z)\}=\Omega_\phi\left\{\sum_{i_1=1}^{k_1} p_{1i_1} z^{g_{1i_1}},...,\sum_{i_n=1}^{k_n} p_{ni_n} z^{g_{ni_n}}\right\} \\ &=\sum_{i_1}^{k_1}\sum_{i_2}^{k_2}\cdots\sum_{i_n}^{k_n}\left(\prod_{j=1}^{n} p_{ji_j} z^{\phi(g_{1i_1},...,g_{ni_n})}\right) \end{aligned} \tag{6}$$

Based on the relationship between MSS performance and the demand level ω that is often determined outside the system, the state space of a MSS can be divided into two subsets: acceptable and unacceptable. The relationship usually is determined by the system state adequacy index r_i defined by $r_i=g_i-\omega$. As a result, state i is acceptable if $r_i \ge 0$. The availability of a MSS (reliability of a non-repairable MSS) is defined as the probability the system staying in the subset of acceptable states. Thus, based on the demand level ω the availability of a MSS, $A(\omega)$, is usually defined as the probability the MSS performance rate is greater than ω [12]. In other words,

$$A(\omega)=\sum_{r_i \ge 0} p_i \tag{7}$$

Then, using operator δ_A it becomes

$$A(\omega)=\delta_A(U(z),\ \omega)=\delta_A\left(\sum_{i=1}^{K} p_i z^{g_i},\ \omega\right)=\sum_{i=1}^{K} p_i \alpha_i$$

where

$$\alpha_i=\begin{cases} 1, & r_i \ge 0 \\ 0, & r_i < 0. \end{cases} \tag{8}$$

In Eq. (8) δ_A is known as UGF operator. This operator determines the polynomial UGF for a group of components first connected in parallel in a subsystem and then for a group of subsystems in series using simple algebraic operations on the individual UGF of components. In some cases, composition operators can be developed for structures with more complex system structure, such as bridges, as shown by Levitin and Lisnianski [8].

3. The recursive algorithm

Li and Zuo [10] presented a recursive algorithm in order to evaluate reliability of multi-state weighted k-out-of-n:G systems. In this type of systems, each component is classified to work in different states and the system is working until sum of the weight of the safe components is at least k. Li and Zuo [10] showed that their method is capable to calculate system reliability in less CPU time compared to UGF.

The notations used in Li and Zuo [10] in their method are:

n: The number of components in each system.
M: The highest possible state of each component and system.
w_{ij}: The weight of component i when it is in state j.