Energy Reports 4 (2018) 393–399

Contents lists available at ScienceDirect

Energy Reports

journal homepage: www.elsevier.com/locate/egyr

Research paper

Energy transfer of Jeffery–Hamel nanofluid flow between non-parallel walls using Maxwell–Garnetts (MG) and Brinkman models

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ARTICLE INFO

Article history:
Received 17 April 2018
Received in revised form 14 May 2018
Accepted 20 May 2018

Keywords:
Energy transfer
Nanofluid
Galerkin method
Jeffery–Hamel flow

ABSTRACT

In this letter, energy transfer of Jeffery–Hamel nanofluid flow in non-parallel walls is investigated analytically using Galerkin method. The effective thermal conductivity and viscosity of nanofluid are calculated by the Maxwell–Garnetts (MG) and Brinkman models, respectively. The influence of the nanofluid volume friction, Reynolds number and angle of the channel on velocity and temperature profiles are investigated. Results show that Nusselt number increases with increase of Reynolds number and nanoparticle volume friction. Also it can be found that skin friction coefficient is an increasing function of Reynolds number, opening angle and nanoparticle volume friction.

1. Introduction

Nanotechnology suggests new kind of working fluid with higher thermal conductivity. Nanofluid can be used in various field of engineering. Fluid heating and cooling are important in many industries fields such as power, manufacturing and transportation. Effective cooling techniques are absolutely needed for cooling any sort of high energy device. Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. In contrast, metals thermal conductivities are up to three times higher than the fluids, so it is naturally desirable to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. Zin et al. (2017) investigated Jeffrey nanofluid free convection in a porous media under the effect of magnetic field. Abro and Khan (2017) investigated flow and heat transfer of Casson fluid in a porous medium. Sheikholeslami et al. (2018a) utilized nanoparticles for condensation process. They analyzed entropy generation and exergy loss of nano-refrigerant. Ullah et al. (2017) investigated slip effect on Casson fluid flow over a porous plate in existence of Lorentz forces. Sheikholeslami et al. (2018f) investigated exergy loss analysis for nanofluid forced convection heat transfer in a pipe with modified turbulators. Sheikholeslami et al. (2018d) presented nanofluid forced convection turbulent flow in a pipe. Sheikholeslami et al. (2018g) studied the nanofluid natural convection in a porous cubic cavity by means of Lattice Boltzmann method. Sheikholeslami (2018e) simulated solidification process of nano-enhanced PCM in a thermal energy storage.

There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square are examples of the WRMs. Hosseini et al. (2018) utilized Galerkin method to investigated Nanofluid heat transfer analysis in a microchannel heat sink (MCHS) under the effect of magnetic field. Vaferi et al. (2012) have studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami (2010) used Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently Least square method is introduced by A. Aziz and M.N. Bouaziz (Bouaziz and Aziz, 2010) and is applied for a predicting the performance of a longitudinal fin Aziz and Bouaziz (2011). They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan (2008) developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations.

After introducing the problem of the flow of fluid through a divergent channel by Jeffery (Sheikholeslami et al., 2018f) and Hamel (1916) in 1915 and 1916, respectively, it is called Jeffery–Hamel flow. On the other hand, the term of Magneto hydro dynamic (MHD) was first introduced by Alfvén (Bansal, 1994) in 1970. The

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https://doi.org/10.1016/j.egyr.2018.05.003
2352-4847/

Nomenclature

A^*	Constant parameter
B_0	Magnetic field(wb.m^{-2})
$f(\eta)$	Dimensionless velocity
Pressure term	Pressure term
Re	Reynolds number
r, θ	Cylindrical coordinates
U_{max}	Maximum value of velocity
u, v	Velocity components along x, y axes, respectively

Greek symbols

α	Angle of the channel
η	Dimensionless angle
θ	Any angle
ρ	Density
ϕ	Nanoparticle volume fraction
μ	Dynamic viscosity
υ	Kinematic viscosity

Subscripts

∞	Condition at infinity
Nanofluid	nf
Base fluid	f
s	Nano-solid-particles

theory of Magneto hydro dynamics is inducing current in a moving conductive fluid in presence of magnetic field; such induced current results force on ions of the conductive fluid. The theoretical study of (MHD) channel has been a subject of great interest due to its extensive applications in designing cooling systems with liquid metals, MHD generators, accelerators, pumps and flow meters (Cha et al., 2002). In recent years, nanofluid has been used in various fields (Sheikholeslami and Shehzad, 2018a; Sheikholeslami et al., 2018b; Sheikholeslami and Rokni, 2018c, a; Sheikholeslami and Shehzad, 2018c; Chamkha et al., 2010; Mansour et al., 2010; Sheikholeslami and Seyednezhad, 2018; Sheikholeslami et al., 2018e; Sheikholeslami, 2018c, a; Chamkha and Ahmed, 2011; Raju and Sandeep, 2016; Sheikholeslami and Rokni, 2018b; Sheikholeslami, 2018b; Sheikholeslami and Shehzad, 2018b; Ali et al., 2016a, b, 2017; Imran et al., 2017; Jafaryar et al., 2018; Sheikholeslami, 2018d; Sheikholeslami et al., 2018c; Sheikholeslami and Sadoughi, 2018; Sheikholeslami and Rokni, 2017b; Fengrui et al., 2017a, b; Sheikholeslami and Seyednezhad, 2017a; Sheikholeslami et al., 2017; Sheikholeslami and Shehzad, 2017a; Sheikholeslami and Rokni, 2017c; Sheikholeslami, 2017c; Sheikholeslami and Shehzad, 2017b; Sheikholeslami and Sadoughi, 2017; Sheikholeslami and Zeeshan, 2017; Sheikholeslami, 2017b; Ahmed et al., 2017; Khan et al., 2017; Sheikholeslami and Bhatti, 2017; Sheikholeslami, 2017a; Sheikholeslami and Seyednezhad, 2017b; Shah et al., 2018; Sheikholeslami and Rokni, 2017a; Sheikholeslami and Ghasemi, 2018).

In this study, the purpose is to solve nonlinear equations through the GM. The effect of active parameters such as nanoparticle volume friction, opening angle and Reynolds number on velocity and temperature boundary layer thicknesses have been examined.

2. Problem description

Consider a system of cylindrical polar coordinates (r, z, θ) which steady two-dimensional flow of an incompressible conducting viscous fluid from a source or sink at channel walls lie in planes, and intersect in the axis of z. Assuming purely radial motion which means that there is no change in the flow parameter along the z direction. The flow depends on r and θ (see Fig. 1).

The reduced forms of continuity, Navier–Stokes and energy equations are (Sheikholeslami et al., 2012):

$$\frac{\rho_{nf}}{r}\frac{\partial\,(ru(r,\theta))}{\partial r}(ru(r,\theta)) = 0, \tag{1}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r} = -\frac{1}{\rho_{nf}}\frac{\partial P}{\partial r} + \upsilon_{nf}\left[\frac{\partial^2 u(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial\theta^2} - \frac{u(r,\theta)}{r^2}\right], \tag{2}$$

$$\frac{1}{\rho_{nf} r}\frac{\partial P}{\partial\theta} - \frac{2\upsilon_{nf}}{r^2}\frac{\partial u(r,\theta)}{\partial\theta} = 0, \tag{3}$$

$$\begin{aligned}(\rho C_p)_{nf}u(r,\theta)\frac{\partial T(r,\theta)}{\partial r} = \\ k_{nf}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(r,\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T(r,\theta)}{\partial\theta^2}\right] \\ +\mu_{nf}\left[2\left(\left(\frac{\partial u(r,\theta)}{\partial r}\right)^2 + \left(\frac{u(r,\theta)}{r}\right)^2\right) + \left(\frac{1}{r}\frac{\partial u(r,\theta)}{\partial r}\right)^2\right],\end{aligned} \tag{4}$$

Where, u_r is the velocity along radial direction, P is the fluid pressure, υ_{nf} the coefficient of kinematic viscosity and ρ_{nf} the fluid density. The effective density ρ_{nf}, the effective dynamic viscosity μ_{nf} and kinematic viscosity υ_{nf} of the nanofluid are given as:

$$\begin{aligned}&\rho_{nf} = \rho_f(1-\phi) + \rho_s\phi, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \\ &\upsilon_{nf} = \frac{\mu_f}{\rho_{nf}}, \quad \frac{k_{nf}}{k_f} = \frac{(k_s+2k_f) - 2\phi(k_s-k_f)}{(k_s+2k_f) + \phi(k_s-k_f)}\end{aligned} \tag{5}$$

Here, ϕ is the solid volume fraction. Considering $u_\theta = 0$ for purely radial flow, one can define the velocity parameter as:

$$f(\theta) = ru_r \tag{6}$$

Introducing the $\eta = \frac{\theta}{\alpha}$ as the dimensionless degree, the dimensionless form of the velocity parameter can be obtained by dividing that to its maximum value as:

$$F(\eta) = \frac{f(\theta)}{u_c}, \frac{T}{T_w} = \frac{\Theta(\theta)}{r^2}, \eta = \frac{\theta}{\alpha}, \tag{7}$$

Substituting Eq. (6) into Eqs. (2) and (3), and eliminating P, one can obtain the ordinary differential equation for the normalized function profile as:

$$f'''(\eta) + 2\alpha\mathrm{Re}[(1-\phi) + \frac{\rho_s}{\rho_f}\phi] \times (1-\phi)^{2.5} f(\eta)f'(\eta) + 4\alpha^2 f'(\eta) = 0, \tag{8}$$

$$\Theta''(\eta) + 4\alpha^2\Theta(\eta) + \frac{(1-\phi) + \frac{(\rho c_p)_s}{(\rho c_p)_f}\phi}{\frac{(k_s+2k_f)-2\phi(k_s-k_f)}{(k_s+2k_f)+\phi(k_s-k_f)}} 2\alpha^2\,\mathrm{Pr}\, f(\eta)\Theta(\eta) + \frac{\mathrm{PrEc}}{\mathrm{Re}\frac{(k_s+2k_f)-2\phi(k_s-k_f)}{(k_s+2k_f)+\phi(k_s-k_f)}(1-\phi)^{2.5}}(4\alpha^2 f(\eta)^2 + f'(\eta)^2) = 0, \tag{9}$$

Where Re is Reynolds number, Pr is Prandtl number and Ec is the Eckert number. On introducing the following non-dimensional quantities,

$$\mathrm{Re} = \frac{f_{\max}\alpha}{\upsilon_f} = \frac{U_{\max} r\alpha}{\upsilon_f} \times \begin{pmatrix} divergent-channel: \alpha > 0, f_{\max} > 0 \\ convergent-channel: \alpha < 0, f_{\max} < 0 \end{pmatrix} \tag{10}$$