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An efficient analytical Bayesian method for reliability and system response updating based on Laplace and inverse first-order reliability computations

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ABSTRACT

This paper presents an efficient analytical Bayesian method for reliability and system response updating without using simulations. The method includes additional information such as measurement data via Bayesian modeling to reduce estimation uncertainties. Laplace approximation method is used to evaluate Bayesian posterior distributions analytically. An efficient algorithm based on inverse first-order reliability method is developed to evaluate system responses given a reliability index or confidence interval. Since the proposed method involves no simulations such as Monte Carlo or Markov chain Monte Carlo simulations, the overall computational efficiency improves significantly, particularly for problems with complicated performance functions. A practical fatigue crack propagation problem with experimental data, and a structural scale example are presented for methodology demonstration. The accuracy and computational efficiency of the proposed method are compared with traditional simulation-based methods.

1. Introduction

Probabilistic inference for system reliability and responses has drawn extensive attentions due to the increasing complexity of modern engineering structures [1,2]. For high reliability demanding systems such as aircraft and nuclear facilities, time-dependent reliability degradation and performance prognostics must be quantified in order to prevent system failures. Reliable predictions of system reliability and system responses are usually required for decision-making in a time and computational resource constrained situation. The central idea of time-independent component reliability analysis involves computation of a multi-dimensional integral over the failure domain of the performance function [3–5]. For problems with high-dimensional parameters, the exact evaluation of this integral is either analytically intractable or computationally infeasible with a given time constraint. Analytical approximations and numerical simulations are two major computational methods to solve this problem [6].

The simulation-based method includes direct Monte Carlo (MC) [7], Importance Sampling (IS) [8,9], and other MC simulations with different sampling techniques. Analytical approximation methods, such as first- and second-order reliability methods (FORM/SORM) have been developed to estimate the reliability without large numbers of MC simulations. FORM and SORM computations are based on linear (first-order) and quadratic (second-order) approximations of the limit-state surface at the *most probable point* (MPP) [3,4]. Under the condition that the limit-state surface at the MPP is close to its linear or quadratic approximation and that no multiple MPPs exist in the limit-state surface, FORM/SORM are sufficiently accurate for engineering purposes [10–13]. If the final objective is to calculate the system response given a reliability index, the inverse reliability method can be used. The most well-known approach is inverse FORM method proposed in [14–16]. Du et al. [17] proposed an inverse reliability strategy and applied it to the integrated robust and reliability design of a vehicle combustion engine piston. Saranyasoontorn and Manuel [18] developed an inverse reliability procedure for wind turbine components. Lee et al. [19] used the inverse reliability analysis for reliability-based design optimization of nonlinear multi-dimensional systems. Cheng et al. [20] presented an artificial neural network based inverse FORM method for solving problems with complex and implicit performance functions. Xiang and Liu [21] applied the inverse FORM method to S-N fatigue life predictions.

Conventional forward and inverse reliability analysis is based on the existing knowledge about the system (e.g., underlying physics, distributions of input variables). Time-dependent reliability degradation and system response changes are not reflected.

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For many engineering problems, usage monitoring or inspection data are usually available at a regular time interval either via structural health monitoring system or non-destructive inspections. The new information can be used to update the initial estimate of system reliability and responses. The critical issue is how to incorporate the existing knowledge and new information into the estimation. Bayesian updating is the most common approach to incorporate these additional data. By continuous Bayesian updating, all the variables of interest are updated and the inference uncertainty can be significantly reduced, provided the additional data are relevant to the problem and they are informative. Hong [22] presented the idea of reliability updating using inspection data. Papadimitriou et al. [23] reported a reliability updating procedure using structural testing data. Graves et al. [24] applied the Bayesian analysis for reliability updating. Wang et al. [25] used Bayesian reliability updating for aging airframe. A similar updating approach using maximum relative entropy principles has also been proposed in [26]. In those studies, Markov chain Monte Carlo (MCMC) simulations have been extensively used. For practical problems with complicated performance functions, simulations are time-consuming and efficient computations are critical for time constrained reliability evaluation and system response prognostics. In structural health management settings, simulation-based method may be infeasible because updating is frequently performed upon the arrival of sensor data. All these applications require efficient and accurate computations. However, very few studies are available on the investigation of complete analytical updating and estimation procedure without using simulations.

The objective of this study is to develop an efficient analytical computational framework for system reliability and response updating without using simulations. Computational components evolved in this approach are Bayesian updating, reliability estimation, and system response estimation given a reliability index. For Bayesian updating, Laplace method [27] is proposed to obtain an analytical representation of the posterior distribution and avoid using simulations. Once the analytical posterior distribution is obtained, the FORM method can be applied to estimate the updated system reliability or probability. In addition, system response predictions given a reliability index or confidence interval can also be updated using the inverse FORM method.

The paper is organized as follows. First, a general Bayesian posterior model for uncertain variables is formulated. Relevant information such as response measures and usage monitoring data are used for updating. Then an analytical approximation to the posterior distribution is derived based on the Laplace method. Next, FORM method is introduced to evaluate the reliability and a simplified algorithm based on inverse FORM method is formulated to calculate system response given a reliability index or confidence interval. Following this, a fatigue crack example with experimental data and a structure scale problem are presented to demonstrate the method. The efficiency and accuracy of the proposed method are compared with traditional simulation-based methods.

2. Probabilistic modeling and Laplace approximation

In this section, a generic posterior model for uncertain parameters is formulated using Bayes' theorem to incorporate additional information such as measurement data. Uncertainties from model parameters, measurement, and mechanism modeling are explicitly included. Laplace approximation is derived to obtain an analytical representation of the posterior distribution. The updated reliability and system responses can readily be evaluated using this posterior distribution.

2.1. Bayesian modeling for uncertain parameters

Consider a general parameterized model $\mathcal{M}(y;x)$ describing an observable event d, where x is an uncertain model parameter vector that need to be updated and y is a model independent variable. If the model is perfect, one obtains $\mathcal{M}(y;x)=d$. In reality, such a perfect model is rarely available due to uncertainties from the simplification of the actual complex physical mechanisms, statistical identification of model parameter x, and the measurement noise in d.

Given a prior probability distribution of x, $p(x|\mathcal{M})$, and the known relationship (conditional probability distribution or likelihood function) between d and x, $p(d|x,\mathcal{M})$, the posterior probability distribution $p(x|d,\mathcal{M})$ is expressed using Bayes' theorem as

$$p(x|d,\mathcal{M}) = p(x|\mathcal{M})p(d|x,\mathcal{M})\frac{1}{\mathcal{Z}} \propto p(x|\mathcal{M})p(d|x,\mathcal{M}), \tag{1}$$

where $\mathcal{Z}=\int_X p(x|\mathcal{M})p(d|x,\mathcal{M})\,\mathrm{d}x$ is the normalizing constant.

The model $\mathcal{M}$ is assumed to be the only feasible model and $\mathcal{M}$ is omitted hereafter for simplicity. Let m be the model prediction and e the error component (for example, the measurement uncertainty of d). The variable d reads

$$d = m+e. \tag{2}$$

The probability distribution for m is represented by the function $p(m|x)=f_M(m)$ and the probability distribution for e is represented by the function $p(e|x)=f_E(e)$. The conditional probability distribution of $p(d|x)$ can be obtained by marginalizing the joint probability distribution of $p(d,m,e|x)$ as follows:

$$p(d|x) = \int_M\int_E p(m|x)p(e|x)p(d,m,e|x)\,\mathrm{d}e\,\mathrm{d}m. \tag{3}$$

Because $d=m+e$,

$$p(d,z,e|x) = \delta(d-m-e). \tag{4}$$

Substitute Eq. (4) into Eq. (3) to obtain

$$p(d|x) = \int_M f_M(m)f_E(d-m)\,\mathrm{d}m. \tag{5}$$

Next, terms $f_M(m)$ and $f_E(e)$ need to be determined. Consider a general case where the model prediction m has a statistical noise component $\epsilon\in\mathcal{E}$ with a distribution function $p(\epsilon|x)=f_{\mathcal{E}}(\epsilon)$ due to the modeling error $m=\mathcal{M}(y;x)+\epsilon$. Eq. (2) is revised as

$$d = \mathcal{M}(y;x)+\epsilon+e. \tag{6}$$

Marginalizing $p(m|\epsilon,\theta)=\delta(m-\mathcal{M}(y;x)-\epsilon)$ over ϵ to obtain

$$f_M(m) = p(m|x) = \int_{\mathcal{E}} p(\epsilon|x)p(m|x,\theta)\,\mathrm{d}\epsilon = f_{\mathcal{E}}(m-\mathcal{M}(y;x)). \tag{7}$$

Eq. (6) is not a model mathematical expectation plus error terms defined in classical regression analysis. $\mathcal{M}(y;x)$ is a general notation for the model and Eq. (6) separates the uncertainty from intrinsic uncertainty (i.e., x), modeling uncertainty (i.e., ϵ) and the measurement uncertainty (i.e., e) for probabilistic modeling. For the purpose of illustration, ϵ and e are assumed to be two independent zero-mean normal variables with standard deviations of σ_ϵ and σ_e, respectively. Eq. (5) is the convolution of two normal distributions and it can be further reduced to another normal distribution as

$$p(d|x) = \frac{1}{\sqrt{2\pi\sigma_{\mathcal{M}}^2}}\exp\left\{\frac{-[d-\mathcal{M}(y;x)]^2}{2\sigma_{\mathcal{M}}^2}\right\}, \tag{8}$$

where $\sigma_{\mathcal{M}}^2=\sigma_\epsilon^2+\sigma_e^2$. If the measurement uncertainty is known (e.g., through calibration test), σ_e can be explicitly assigned.

Substituting Eq. (8) into Eq. (1) yields the posterior probability distribution of the uncertain parameter x incorporating the observable event d. The reliability and responses of the system can