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Redundancy of structural systems with and without maintenance: An approach based on lifetime functions

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ABSTRACT

The lifetime reliability of existing structures may be quantified by lifetime functions. Redundancy is an additional type of structural performance indicator that is defined as a measure of warning available prior to system collapse. Lifetime functions provide a basis on which lifetime redundancy can be evaluated and its quantification can be formulated. The objective of this paper is to present a novel approach for the evaluation of the lifetime redundancy of structural systems. Measures of lifetime redundancy based on lifetime functions are investigated. The effects of maintenance on lifetime functions and redundancy are also presented. Furthermore, the lifetime redundancy is incorporated in a maintenance optimization algorithm. This optimization algorithm is illustrated on an existing highway bridge.

1. Introduction

It is reported by the Federal Highway Administration [14] that as of the end of the year 2007, over a quarter of the estimated 600,000 highway bridges in the United States were either structurally deficient or functionally obsolete. This alarming fact draws strong attention to the need of seeking measures to improve the status of this bridge inventory. The scarcity of funds available dictates that these measures are optimally effective and economical. These facts shifted the interest of many researchers over the last decade to the area of assessment and prediction of the structural performance of highway bridges and optimum planning of maintenance interventions.

The efforts of researchers in this field have offered a variety of models and techniques that aim to accurately assess the current condition and predict the future changes of the structural performance of highway bridges. Today, it is still a paramount interest of the researchers in this field to either improve existing models or propose new models that provide efficiency and economy.

Research on the topic of using lifetime functions has shown their effectiveness as useful tools in quantifying the lifetime reliability of highway bridges [16]. Lifetime functions are mathematical models representing the time-variant performance of structural components and systems. Optimum maintenance strategies have been obtained based on the predicted lifetime reliability [17]. However, lifetime functions offer the ability of deriving a number of other performance indicators, one of which is the lifetime redundancy.

One of the lessons that should be learned from the failure of several structures such as the failure of the I-35W Mississippi River Bridge in August 2007 is the crucial need of providing redundancy in structures. Although, the lack of redundancy was not apparently the direct cause of the collapse of the I-35W Bridge, it may be argued that additional load paths may have prevented that event. Even a single flaw in the design or construction of a structure may turn into a deadly catastrophe. Alternate load paths provided by redundant members may prevent the occurrence of such a catastrophe. Hence, predicting and maintaining the lifetime redundancy, in addition to the lifetime safety, is a desired goal of an improved maintenance optimization algorithm.

This paper investigates possible measures for the lifetime redundancy of structural systems using representations of lifetime functions. The effects of maintenance on lifetime functions and redundancy are also presented. Furthermore, the lifetime redundancy is incorporated into a maintenance optimization algorithm. This optimization algorithm is illustrated on an existing highway bridge.

2. Lifetime component reliability measures

Lifetime functions offer a number of lifetime reliability measures for components and systems. A number of these

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measures are introduced in this section. These measures have been used extensively by many structural reliability researchers for all types of structures including bridges. This section is a background review of these measures. The discussion in this section is limited to components that are not maintained or repaired throughout their service life. This limitation, however, will be removed in later sections.

2.1. Time to failure probability density function

The probability density function (PDF) of the time to failure is the link between available statistical information and the predictive lifetime models. The time to failure of a component, treated as a random variable T, is defined as the time elapsing from the time the component is put into operation until it fails for the first time [6]. The choice of the time to failure PDF is dictated by the component characteristics and its failure pattern. Typical time to failure PDFs include the Weibull distribution and the exponential distribution, which is a special case of the Weibull distribution. In fact, a Weibull distribution has been fitted to the lifetimes of the Dutch stock of concrete bridges [15]. The PDF of the Weibull distribution is defined as [9]

$$f(t) = \begin{cases} \kappa\lambda(\lambda t)^{\kappa-1}e^{-(\lambda t)^{\kappa}} & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where λ is a scale parameter and κ is a shape parameter. The exponential function is a special case of the Weibull function having a shape parameter κ=1.0.

2.2. Survivor function and cumulative probability of failure

The cumulative probability of failure $F(t)$ is the probability that the time to failure of a component is less than t and is calculated as

$$P(T \le t) = \int_0^t f(u)\,du \quad (2)$$

The complement of $F(t)$ is the survivor function $S(t)$, sometimes referred to as the reliability function, which is the probability that the component will not fail before time t and is calculated as [9]

$$S(t) = 1 - F(t) = P(T > t) = \int_t^{\infty} f(u)\,du \quad (3)$$

Fig. 1 illustrates schematically how $F(t)$, $S(t)$ and $f(t)$ are geometrically interrelated. The area under $f(t)$ is 1.0 and it is divided into the area before the time t_f that represents $F(t_f)$ and the area after the time t_f that represents $S(t_f)$. It is worth emphasizing that the time to failure PDF, $f(t)$ can also be derived from its survivor function as follows [9]:

$$f(t) = \frac{-dS(t)}{dt} = \frac{dF(t)}{dt} \quad (4)$$

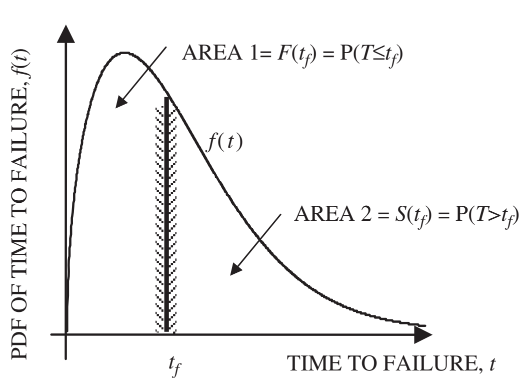

Fig. 1. Geometric relationship between the PDF of the time to failure $f(t)$, the survivor function $S(t)$; and the cumulative probability of failure $F(t)$.

2.3. Hazard function and cumulative hazard function

The hazard function, $h(t)$, also known as the failure rate, is defined as the conditional probability that given a component has survived until time t it will fail in the time interval $t+dt$ [13]. The hazard function is calculated as [9]

$$h(t) = \frac{f(t)}{S(t)} = -\frac{dS(t)}{dt}\frac{1}{S(t)} \quad (5)$$

The cumulative failure rate from the time the component is put in service until the time t is also known as the cumulative hazard function, $H(t)$, which is calculated as [9]

$$H(t) = \int_0^t h(u)\,du \quad (6)$$

2.4. Availability

A component is available at time t if it is functioning at time t. The event that a component survives (i.e., does not fail) up to time t is in fact the same event that the component is available at time t. Therefore, the probability of survival of a component from the time it is put in service until time t is the same as the probability that the component is available at time t. Hence, the availability $A(t)$ is equivalent to the survivor function $S(t)$ of a non-repairable component. In fact, the survivor function of a non-repairable component is a special case of its availability. They both differ when the component is repaired or replaced [8]. This issue will be discussed later. The unavailability of a component, $An(t)=1-A(t)$, is the probability that it has failed before time t and thus it is unavailable (not functioning) at time t [2].

3. Lifetime system reliability

Complex systems are usually decomposed into functional entities composed of components or subsystems for the purpose of reliability analysis. By combining the appropriate series and parallel subsystems of the system model systematically, the entire system can be reduced to one single equivalent component. The reliability of this equivalent component is the reliability of the system. In fact, this method has successfully been applied in structural engineering [17,18].

However, many practical systems have complex structures that are not purely series, parallel or series–parallel [13]. Various more general techniques are available to treat such systems. The minimal cut set method is chosen for this study.

The first step in the minimal cut set method is to identify all cut or path sets of the system. A minimal cut set is a set of components in the system which by failing causes the system to fail, but if does not fail the system does not fail. On the other hand, a minimal path set is a set of components in the system which by functioning ensures the system to function, but if fails the system fails [6]. The second step is to establish the state function of the system. The state function of a system composed of n components is a binary function described as [6]

$$\phi(\mathbf{x}) = \phi(x_1, x_2, \ldots, x_n) \quad (7)$$

and takes a value of 1.0 if the system is functioning and 0.0 otherwise, where x_i is a binary variable that describes the state of the component i such that it takes the value of 1.0 if the component is functioning and 0.0 otherwise. The state function of