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Uncertainty analysis using evidence theory – confronting level-1 and level-2 approaches with data availability and computational constraints

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ABSTRACT

Dempster–Shafer Theory of Evidence (DST), as an alternative or complementary approach to the representation of uncertainty, is gradually being explored with complex practical applications beyond purely algebraic examples. This paper reviews literature documenting such complex applications and studies its applicability from the point of view of the nature and amount of data that is typically available in industrial risk analysis: medium-size frequential observations for aleatory components, small noised datasets for model parameters and expert judgment for other components. On the basis of a simple flood model encoding typical risk analysis features, different approaches to quantify uncertainty in DST are reviewed and benchmarked in that perspective: (i) combining all sources of uncertainty under a single-level DST model; (ii) separating aleatory and epistemic uncertainties, respectively, modeled with a first probabilistic layer and a second one under DST. Methods for handling data in probabilistic studies such as Kolmogorov–Smirnov tests and quantile–quantile plots are transferred to the domain of DST. We illustrate how data availability guides the choice of the settings and how results and sensitivity analyses can be interpreted in the domain of DST, concluding with recommendations for industrial practice.

1. Introduction

Dempster–Shafer Theory of Evidence (DST) as an alternative method for the representation of uncertainty has gained an increasing amount of attention both from the theoretical and the applied point of view. DST is still a young field compared to probabilistic analysis with the first major works published by [1,2]. The main focus on early DST approaches was on data fusion and artificial intelligence. The ability of DST to model and propagate uncertainty through systems was only sparsely utilized in the last millennium. However, in the recent past, several approaches favouring DST have emerged and drive on the development of this theory towards larger applicability. A turn point may have come with the Sandia workshop on epistemic uncertainty, whose results culminated in a special issue of this journal [3]. Beyond the initial algebraic benchmarks, the present paper is motivated by the ongoing discussion on the applicability and modeling best practices of DST for complex industrial risk models.

As will be reviewed by this paper, a number of challenges come up with the use of DST within complex industrial risk analyses: the complexity of communicating results to various stake-holders, of encoding it within formal regulatory processes [4], or the computational load [5,6]. This paper then focuses on an additional challenge: the issue of the nature and amount of data that is typically available in industrial risk analysis, and the practicality of various DST or combined probabilistic-DST settings to represent such information. In this work, we illustrate on a representative risk analysis example different ways and propose new methods to deal with data, expert estimates, propagation, result interpretation and post-processing/sensitivity analysis. We focus on the interpretation and discussion on the aspects that are most relevant to the practitioner.

Another important focus of this work is the comparison of several settings using DST, discussing the underlying paradigms and applying recent techniques for uncertainty modeling and propagation. The differences in the uncertainty paradigms of a level-1 and a level-2 DST approach will be discussed and compared to probabilistic approaches. Hereafter, level-1 refers to approaches eschewing any distinction between kinds of uncertainty, and hence randomise them altogether within a combined sample space. Level-2 refers to approaches separating aleatory and epistemic components into hierarchic uncertainty models, and hence involves more elaborate sample spaces of distributions with distributed parameters.

The remainder of this paper is structured as follows: Section 2 formalizes and compares in more detail the level-1 and level-2 approaches, which utilize probabilistic or DST models. Section 3

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provides a review of recent, industrially relevant DST case studies, focussing essentially on the comparison and categorization with respect to certain criteria. Section 4 presents an educational example in the field of hydrology, developed as a realistic benchmark for uncertainty modeling throughout the French industry. Section 5 builds level-1 and level-2 uncertainty models from available data and expert knowledge. Section 6 addresses the interpretation of results that are obtained when propagating level-1 and level-2 models through the system function and discusses the possibilities for post-processing results. The article ends with a critical discussion of the study results and the advantages and drawbacks of the different quantifications.

2. The issue of uncertainty modeling and alternative frameworks

2.1. The need for a choice of uncertainty representation

Uncertainty studies involve the representation of the uncertainty affecting the inputs $\mathbf{x}=(x_1,\ldots,x_p)$, $x\in\Omega$ of a system model f and the study of its impact on the output variable(s) of interest z

$$z=f(x_1,\ldots,x_p) \tag{1}$$

Within the field of industrial risk analysis, x is thought to cover a large variety of parameters, such as system characteristics, dimensions, material properties, operating conditions, state of system components or initiating events or even model parameters. The variable(s) of interest z is (are) usually incorporating safety or performance indicators. Components of x may be affected by different types of uncertainty sources such as inner variability (space, time), lack of knowledge, model imperfections, etc.

Uncertainty treatment involves the choice of a mathematical representation of uncertainty or setting, which may be deterministic, probabilistic or non-probabilistic. Such a setting takes the form of a *distribution* over a sample space Ω of possible values, the mathematical definition of which depends on the paradigm chosen as will be detailed subsequently.

Hence the classical steps [7] are: uncertainty modeling (i.e. the choice of a distribution representing the extent of uncertainty in the inputs), uncertainty propagation (i.e. the computation of the extent of uncertainty in the outputs) and ranking/sensitivity analysis (i.e. apportioning relative input contributions into output uncertainty). The final outcome is an estimated uncertainty model on the variable(s) of interest and associated *quantities of interest*: according to the goal of the study, these can be the measures of exceeding a threshold, mean/median of the output uncertainty, a measure of dispersion or information on quantiles.

The most frequent representation of uncertainty is the standard (or level-1) probabilistic setting, wherein a probability distribution defined over Ω represents x as a random variable. Two popular interpretations of such representation have been extensively discussed in [8–13]. The frequentist interpretation considers x and $z=f(x)$ as observable realizations of an underlying, repeatable probabilistic model and this underlying probabilistic model can be approximated from a large set of data. The subjective interpretation considers probability distributions as subjective preferences of the decision-maker [7,14], without the need for an underlying repeatable phenomenon with observable frequencies. In using a level-1 probabilistic setting, the user refers either to the frequentist or the subjective perspective. In both cases, common quantities of interest are e.g. $P(Z>c)$, the probability of exceeding a threshold c or the 99% quantile $Q99(Z)$. An often-criticized drawback is the lack of separate accounting for aleatory and epistemic uncertainty, which is not obtainable within the level-1 setting. Several alternative ways have been developed for such situations, including DST or double-level probabilistic settings reviewed here below.

2.2. Uncertainty representation in evidence theory

This section serves a short introduction into DST as the method is not as well known as probability theory. Comprehensive works with detailed presentations of DST are e.g. [15–17].

A good way to introduce DST is to present the differences compared to probabilistic modeling, and this will be done in discrete (countable) spaces to keep simple. Uncertainty regarding a discrete random variable x in a probabilistic model is represented by a random variable X. This means that a mass function m can be used to describe the evidence put on each value $x\in\Omega$, expressing the probability $P(X=x)$. This mass function is given as

$$\begin{aligned} &m:\Omega\to[0,1]\\ &\sum_{a\in\Omega} m(a)=1 \end{aligned} \tag{2}$$

In discrete probabilistic models, m defines a probability distribution and $P(X=x)=m(x)$. It is important to outline this well-known equality, because it will not hold in the case of DST. From m, the probability of x to be in interval $[\underline{x},\overline{x}]$, $P(x\in[\underline{x},\overline{x}])$ can be obtained by adding over all mass values

$$P(x\in[\underline{x},\overline{x}])=\sum_{a\in[\underline{x},\overline{x}]} m(a) \tag{3}$$

The cumulative distribution function (CDF) $F(x):=P(x\in[-Inf,x])$ is a special case of Eq. (3).

More formally, probability theory involves developing over the sample space Ω a probability space (Ω,A,P), where the probability function P is defined on a suitably restricted collection $A=(A_\lambda)_\lambda$ of subsets (events) A_λ of Ω that enjoys the properties of a σ-algebra, i.e. contains $\varnothing$, Ω and is stable through the countable elementary operations of complementation, union and intersection as follows:

$$A_\lambda c\mathcal{A}=\Omega\backslash A_\lambda c\mathcal{A} \tag{4}$$

$$(A_\mu)_\mu c\mathcal{A}=U_\mu(A_\mu)c\mathcal{A} \tag{5}$$

the combination of those two obviously implying stability under countable intersections. Hence probability, as a function mapping such collection of events A into [0, 1] is required to respect $P(\varnothing)=0$ and $P(\Omega)=1$ and to be sub-additive: provided that $A\cap B=\varnothing$ then $P(A\cup B)=P(A)+P(B)$, and such additivity holds for any countable collection of disjoint events $(A_\mu)_\mu$.

Those basic properties are essential to understand the differences with Dempster–Shafer representations. In Dempster–Shafer, a mass function $m:\mathcal{P}(\Omega)\to[0,1]$ maps probability masses on sets, not only on point values. Each subset of Ω with a mass >0 is referred to as focal element. Recall that the descriptions and methods in this work are restricted to mass functions with a finite number of focal elements (i.e. intervals or points with a probability mass). In that case, descriptions on continuous variables but with a finite number of focal elements are similar to discrete distributions except one *crucial* difference. In discrete distributions, probability masses can only be assigned to points. However, in DST, the (probability) mass function not only assigned to single values but instead to sets or ranges. In DST, uncertainty on x is described by its basic probability assignment (BPA) m:

$$\begin{aligned} &m:\mathcal{P}(\Omega)\to[0,1]\\ &m(\varnothing)=0\\ &\sum_{A\subseteq\Omega} m(A)=1 \end{aligned} \tag{6}$$