Theoretical and Applied Fracture Mechanics 71 (2014) 67–75

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

The influence of the first non-singular stress terms on crack initiation direction in an orthotropic bi-material plate

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ARTICLE INFO

Article history:
Available online 11 June 2014

Keywords:
Bi-material notch
Crack initiation direction
Non-singular stress term
Generalized fracture mechanics
Path-independent integral

ABSTRACT

A bi-material plate composed of two orthotropic parts leads to free edge stress singularity. This is considered as a bi-material notch with $\omega_1 = \omega_2 = 90°$ under general loading. Besides the singular term the first non-singular term is also included in the stress distribution at the notch tip. The Stroh–Eshelby–Lekhnitskii formalism for plane elasticity is used to express stress and displacement fields and the strain energy density factor. The exponents of the singular and non-singular stress terms and corresponding eigenvectors are the solution of the eigenvalue problem resulting from the prescribed boundary and compatibility conditions at the notch tip. The potential direction of crack initiation is determined from the local minimum of the mean value of the strain energy density factor in both materials. The necessary knowledge of the generalized stress intensity factor and generalized *T*-stress is realized by means of the employment of the two state conservation integrals. Following the assumption of the same mechanism of rupture in the case of a crack and a notch, an expression for the critical values of the generalized stress intensity factor is obtained.

1. Introduction

The increasing use of fiber reinforced composites has brought a renewed interest in the analysis of cracks, notches and multi-material corners in anisotropic materials. In most of the geometrical and material configurations of a bi-material corner, there are two singular terms in the stress distribution at its tip. Contrary to a crack in a homogenous material or a crack at a bi-material interface, the stress singularity exponents differ from $-1/2$ and the corresponding generalized stress intensity factors belong to both the normal and shear modes of loading. It was shown in [1] that the domain in which the stress singular terms are dominant could be rather small with respect to domains ahead of the other kinds of singularities, e.g. cracks. This is especially apparent in the case of a free edge composed of two orthotropic parts considered in the paper. Hence it is supposed that the orthotropy, see [2], and non-singular terms of the stress distribution near the notch tip play an important role in the estimation of the initiated crack direction and in the notch stability criterion. General singular stress concentrators exhibit the stress distribution at their tip similar to those of a crack as described in linear elastic fracture mechanics. In the same way, the non-singular higher order terms (appearing in the stress distribution of the crack) have their generalized form in cases of general singular stress concentrators.

The advanced studies of linear elastic fracture mechanics of cracks show an influence of particular singular and non-singular stress series terms on the fracture behavior of solids with a crack. It has been shown that *T*-stress plays an important role within crack behavior assessment both in the case of brittle fracture and in the case of fatigue crack propagation [3–5]. Similarly, the effects of *T*-stress on interfacial cracks in isotropic and anisotropic bi-materials were studied [5,6]. In the case of presence of notches, further parameters, such as the influence of the out-of-plane mode induced at the tip of notches, are important, e.g. [7,8]. Combination of the 3D effect and the influence of the non-singular stress terms would be interesting as well.

Depending on loading conditions and the geometry of a component with the stress concentrator, the generalized constraint can have a positive or negative influence. It can counteract crack initiation or it can stimulate it. Thus assessment not covering the influence of the constraint provides overestimated or underestimated results. In the former case the generalized approaches covering singular and non-singular stress terms can save a certain volume of material, while in the latter case the (singular and non-singular) stability assessment can prevent fatal damage.

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http://dx.doi.org/10.1016/j.tafmec.2014.05.008

Nomenclature

$\boldsymbol{A}$, $\boldsymbol{L}$	Suo's notation of the eigenmatrices of the Stroh's formalism
$\boldsymbol{B}$, $\boldsymbol{K}$, $\boldsymbol{Y}$	matrices of the eigenproblem of the notch
C_{ijks}, S_{ijks}	components of the stiffness and compliance tensor
E^*	Young modulus respecting the orthotropy of the materials
$F_H(\theta)$, $F_{HT}(\theta)$, $F_T(\theta)$	angular functions appearing in the expression of the strain energy density factor
H, T	generalized stress intensity factor and T-stress
H_C	critical value of the generalized stress intensity factor
$k =$	$\begin{cases} 1-\nu^* \text{ for plain strain,} \\ (1-\nu^*)/(1+\nu^*) \text{ for plain stress,} \end{cases}$
K_{IC}	fracture toughness
r_0	radius of the integration path Γ
$R_j(\theta)$	radius of the point z_j in the z_* – plane of the Stroh's formalism
s_{ij}, s'_{ij}	components of elastic compliance matrix and their reduced form in the contracted notation
S_C	critical value of the strain energy density factor
$\boldsymbol{t}(\boldsymbol{u}^h)$	tractions along the integration path Γ evaluated via the finite element method
$\boldsymbol{T}$	resulting force expressed along the half-line
$\boldsymbol{u}(r,\theta)$	vector of the regular displacements
$\hat{\boldsymbol{u}}(r,\theta)$	vector of the auxiliary displacements
$\boldsymbol{u}^h$	displacements along the integration path Γ evaluated via the finite element method
υ_H, υ_T	eigenvectors of the functions $\boldsymbol{\eta}_H$, $\boldsymbol{\eta}_T$ and $\boldsymbol{\lambda}_H$, $\boldsymbol{\lambda}_T$
$\hat{\boldsymbol{\upsilon}}_H, \hat{\boldsymbol{\upsilon}}_T$	eigenvectors of the auxiliary functions $\hat{\boldsymbol{\eta}}_H, \hat{\boldsymbol{\eta}}_T$ and $\hat{\boldsymbol{\lambda}}_H, \hat{\boldsymbol{\lambda}}_T$
V	volume of the material
W	strain energy
$\boldsymbol{Z}_H(\theta)$, $\boldsymbol{Z}_T(\theta)$	angular matrices of the Stroh's formalism
α	direction of the applied loads
Γ	integration circular path
$\delta_H - 1$	stress singularity exponent
$\delta_T - 1$	stress exponent corresponding to the generalized T-stress
$\boldsymbol{\eta}_H(\theta)$, $\boldsymbol{\eta}_T(\theta)$	angular eigenfunction of regular displacements
$\hat{\boldsymbol{\eta}}_H(\theta), \hat{\boldsymbol{\eta}}_T(\theta)$	angular eigenfunction of auxiliary displacements
$\boldsymbol{\lambda}_H(\theta)$, $\boldsymbol{\lambda}_T(\theta)$	angular eigenfunction of the regular stress function
$\hat{\boldsymbol{\lambda}}_H(\theta), \hat{\boldsymbol{\lambda}}_T(\theta)$	angular eigenfunction of the rauxiliary stress function
$\boldsymbol{\Lambda}_{ij}^H(\theta), \boldsymbol{\Lambda}_{ij}^T(\theta)$	angular functions appearing in the expression of the strain energy density factor
μ_j	eigenvalues of the material
σ^{appl}	applied loads
σ_{ij}	components of the stress tenzor
$\Sigma(r,\theta)$	strain energy density factor
$\overline{\Sigma}(r,\theta)$	average strain energy density factor
$\phi = [\varphi_1, \varphi_2]^T$	stress complex vector function
$\Psi_j(\theta)$	argument of the point z_j in the z_*-plane of the Stroh's formalism
$\Psi(\boldsymbol{u}, \hat{\boldsymbol{u}})$	Ψ-integral
ω_1, ω_2	angles of the notch geometry

The crack initiation criteria of generalized stress concentrators require establishing a specific distance from the tip of the concentrator, which depends on material characteristics (strength and fracture toughness of the material [9]) or a size of material grain, [10,11]. As it is mentioned above, unfortunately, these distances are in some cases much larger than the characteristic dimension of the domain where the stress state is controlled by singular terms [12,13]. Consequently, the influence of the non-singular stress terms can be significant and this emphasizes the necessity of their study for deeper understanding of fracture processes at a bi-material notch tip.

If the notch angle becomes zero, the notch turns into a sharp crack and the second term of Williams' solution, often called T-stress, is a constant term independent of the distance from the crack tip. The previous studies on fracture assessment of cracked bodies have shown that in addition to the singular term, T-stress may significantly affect the process of crack growth. For example, in [14,15] the effect of T-stress on the size and shape of the plastic zone in mode I loading is investigated. In [16] it is also shown that T-stress could influence initiation of brittle fracture. Furthermore, it has been shown that in loading mode I T-stress is an important parameter for the stability analysis of the fracture trajectory, [17]. They suggested that for positive values of T-stress, the fracture trajectory gradually deviates from the line of the initial crack. In contrast, specimens with negative T-stress exhibit a stable fracture path. Calculations of non-singular terms were performed using various methods [18–20].

The effects of the non-singular terms on the behavior of cracks have been assessed by many researchers. Despite this, almost no results have been reported concerning the influence of non-singular terms on the stress distribution description around the generalized stress concentrators such as sharp notches or bi-material notches. In [21] the effect of the first non-singular term of mode I is studied with respect to the size and shape of the plastic zone around a sharp notch. The influence of the presence of generalized T-stress on stress distribution in the case of a sharp notch in homogeneous material is studied in [22]. In [23] the photoelasticity method is used to determine the higher order stress terms in bi-material notches. Further, an overdeterministic method is used to calculate the generalized stress intensity factors and the coefficients of the higher order terms for structures containing sharp notches [24] and the first studies of evaluation of the eigenvalues of the first non-singular term for bi-material notches are presented in [25].

The following article deals with the stability criteria and criteria of crack initiation direction of a $\omega_1 = \omega_2 = 90°$ orthotropic bi-material notch by means of the average value of the strain energy density factor [26] over some distance from the notch tip. These results are based on the knowledge of the generalized stress intensity factors and generalized T-stresses, which are evaluated using the analytical–numerical methods. Besides singular and non-singular terms evaluation, these analytical–numerical methods allow us to assess the precision of the obtained results by means of their comparison with purely numerical results.

2. A stress and displacement distribution near the notch tip

The geometry, loading and coordinate system introduced in the studied notch is given in Fig. 1. The problem is solved as a plane one and an ideal interface between the materials of the notch is supposed. Both materials are assumed to be orthotropic. Further the real values of the stress singularity exponents and corresponding generalized stress intensity factor and T-stress are assumed. The planeness of the problem allows us to express the stresses and displacements via analytical expressions. There are several methods providing this possibility, but the Lekhnitskii, Eshelby and Stroh formalism offers an elegant way based on the