Theoretical and Applied Fracture Mechanics 71 (2014) 79–84

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Failure criteria: Old wines in new bottles?

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ARTICLE INFO

Article history:
Available online 12 April 2014

Keywords:
Size effect
Exact solution
Volume–surface strain energy
Failure criteria

ABSTRACT

Aim of the present work is to communicate a part of the experience gathered by the first author after 40 years of involvement in Fracture Mechanics and Failure of Materials in general, through close collaboration with his colleagues. From this point of view, a few prerequisites for the, say, correct formation of a failure criterion are considered. For example, are of equal validity deductive and inductive approaches? Furthermore, some common characteristics of existing failure criteria are discussed and their effect on the quality and the general applicability of failure criteria is presented. For example, how and why does geometry affect failure predictions? Do cracks or other singularities require a special treatment? Is the characterization of materials through usual constitutive equations adequate? In a more practical level, the necessity of introducing as more as possible stress/strain components in the formation of a failure criterion is emphasized, driving directly to strain energy density (SED) considerations. The deterministic requirement of "cause-effect" demands available (i.e. elastic) SED, excluding plastic work, and, consequently, plastic strains from the formation of any criterion. Considering the only two mechanisms of storing SED in materials (volume-lengths and shape-angles changes of the elementary volume), we arrive into a dilemma regarding the "behavior" of the SED parts been spent for volume or shape changes. In case they are collaborative, their sum (i.e. the total elastic SED) is adequate to describe failure. Contrarily, in case of competitive behavior, each SED component has its-own importance and must be traced separately. Finally, existing groups of criteria are commented and some conclusions are presented.

1. Introduction

In the Euclidian–Newtonian world – where we and other materials live – source of knowledge and wisdom is the direct observation of our environment. We put an order/explanation to phenomena by either (a) introducing an axiomatic hypothesis governing a class of them and concluding, in a deductive manner (top down), their observed behavior or (b) comparing many similar cases in order to locate behavioral similarities allowing for the extraction of a rule in an inductive way (bottom up). Both methods are almost equivalent but in case of induction it is not clear if the prerequisite of causality is satisfied. All observed parameters may not play role in the phenomenon, being in fact side-effects. In other words, a series of experiments may result in a dilemma concerning the "*space*" of description of the experimental data, namely which of the parameters are causally connected, the remaining being simply "present". Classical example of a "*wrong*" space is fatigue where a purely linear elastic parameter (stress intensity factor K_I) is used to describe a purely non-linear elastic/plastic strains phenomenon. The outcome, after almost two centuries of hard work, is disappointing. A Keplerian approach is, still, missing to put an order to the Ptolemaic chaos of the world of fatigue. This is due to the fact that causality is not satisfied (K_I is irrelevant to, at least, elastoplastic phenomenon of fatigue and after a few cycles is meaningless) and Paris law for fatigue is in fact an interpolation procedure. Why is causality important? Because the Euclidian–Newtonian world is deterministic and in practical terms, no one can guarantee that an even huge number of experiments is wide enough to cover extreme cases belonging apparently to the same group. Hence the absence of a rational cause drives to a kind of "*conceptual extrapolation*", often accompanying inductive approaches.

On the other hand, deductive approaches fail to predict exactly experimental results, showing however a more or less acceptable agreement with them. In any case, they are free from the dangerous and unpredictable conceptual extrapolation and, with a proper safety factor, can cover a really wide range (if not all) of similar phenomena. In other words, deductive approaches serve as the thin main line passing through experimental data. This way, axiomatic-deductive approaches can be considered as "*Laws*", although inductive ones may be called "*Results*".

The space of description of a class of similar phenomena may be wrong as in case of fatigue but, also, it may be "*incomplete*". The dimensions of such spaces are fewer than the number of

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http://dx.doi.org/10.1016/j.tafmec.2014.03.007
0167-8442/

independent physical parameters being involved. Constitutive equations are a typical example. In the friendliest case of an isotropic material two mechanical properties are necessary to describe its behavior, implying the execution of two independent experiments. Instead, one experiment is executed (usually uniaxial tension to obtain $\sigma_1 = f(e_1)$) assuming identical results for all three axes. The area under this curve represents the total SED. The second independent experiment (usually torsion) is required to evaluate, for example, volume expansion or shear modulus. Following the rationale of von Mises, four "equivalent" quantities (equivalent stress/strain σ_{eq}, e_{eq} and hydrostatic pressure/volume expansion p, Θ are introduced, which in terms of principal stresses/strains are:

$$\left.\begin{aligned}\sigma_{eq} &= \tfrac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}, \quad p=(\sigma_1+\sigma_2+\sigma_3)/3\\ e_{eq} &= \tfrac{1}{\sqrt{2}(1+\upsilon)}\left[(e_1-e_2)^2+(e_2-e_3)^2+(e_3-e_1)^2\right]^{1/2}, \quad \Theta=e_1+e_2+e_3\end{aligned}\right\} \tag{1}$$

They form two constitutive equations, $\sigma_{eq} = f(e_{eq})$ and $p = f(\Theta)$. The first of them, being the classical von Mises equation, describes the behavior of a material under shear stresses *per se* and the second one the respective behavior under normal stresses *per se*. This way, an interpretation of the response of materials in physical terms is obtained and the space of description is complete.

The problem of selection of the correct space of description of a class of experiments becomes more difficult when guidance from the appropriate constitutive equations is absent. One has to make a choice of a pair of mechanical quantities (stresses, strains, SED, plastic work, etc.) between about 170 different combinations of these quantities! Most of them are *a priori* rejected in terms of taste and experience of the researcher. Yet, enough room remains for the introduction of many failure criteria. So, it is rather conservative to conclude that even a single set of objective experimental data can be the launching point of appearance of some dozens of criteria to interpret them. At eighties and nineties, whole issues of reputable journals, like *Int. J. of Fracture*, *Engng. Fracture Mechanics* and many others, were solely devoted to "new" fracture criteria [1]. Beyond the unnecessary saturation of information [2] which forbids even to have a look at most of them, the necessity of introduction of a, so to say, hyper-criterion emerges for the proper selection of one out of dozens "normal" criteria.

An additional cause of confusion is the effect of geometry (size and shape of the specimen) on the failure of materials. Here, by "Geometry" is meant not only the silhouette (external shape) of a specimen but, also, any pre-existing internal flaws. Flaws are loci of geometry (structure) different from that of neighboring areas. A through thickness visible crack works in the same way as an invisible embedded crack. Both redistribute stress field. It is known that the strength of concrete depends on the volume (size) of the specimen even in case of minimal parasitic shear stresses between grips and specimen. Increased volume increases the number of strong microstructural singularities (natural flaws) causing premature failure of the bigger specimen. On the other hand, specimens of the same volume but different shape have different surface/volume ratio. This change is vital for the failure of materials, as we will see in the sequel. Consequently, the apparent "size effect" is caused by the departure of the material from the assumption of geometrical smoothness and continuity of Mechanics of Continua. In case of a large number of flaws with roughly equal severity, "Statistics" of Nature allows for acceptably constant mean behavior of a material. This mean behavior changes when either the same population is partially exposed to different boundary conditions (shape effect) or varies by a few orders of magnitude (size effect). The present task is to describe shape/size effect in pure terms of Mechanics of Continua avoiding the introduction of ill-defined qualitative terms like "core region", "fracture process zone" or categorization of materials as quasibrittle or quasiductile.

The natural inhomogeneity of materials was faced by Griffith through his bright idea of introducing an enormous macroscopic defect, called "crack". The presence of an artificial macro-crack allows for the annihilation of any natural flaws in the specimen. Then, the material becomes a continuous mean with "strange" geometry, allowing for the application of the laws of Mechanics of Continua. The presence of a crack modifies the stress distribution in the specimen but not the laws governing its behavior because stresses are geometrical entities, independent of material properties. This "Iron Rule" is violated only in case of rapidly changing loads, due to the appearance of mass-inertia phenomena. It is difficult to accept that geometry affects the behavior of materials through an additional fictitious mechanism beyond stress redistribution. In that case, we had to accept that specimen geometry is a material property. So, one wonders on the rational basis of different values of critical stress intensity factors between plane-stress and plane-strain geometries. In the sequel, we will expand some of the as above mentioned comments in an attempt to show that novel approaches are not necessarily better approaches.

2. Overview of failure criteria

Let us now make some short comments on existing classes of failure criteria.

2.1. Incomplete space – inductive approach

We start with an example of inductive treatment of a familiar problem in Strength of Materials, namely sheet metalforming. There a Forming Limit Diagram (FLD) is asked to describe limiting strains. This FLD is obtained by either in-plane stretching or out-of-plane (punch) stretching of the sheet. Pairs (e_A, e_B) of such strains seem to follow two empirical rules, namely $e_A \approx 1 + e_B$ or $e_A \approx 1 - 0.5e_B$ (see for example [3].

However, a problem of incomplete space appears. A, through-thickness third strain e_3 exists as obtained from equivoluminal changes of plastic strains. Then, in general we have:

$$\left.\begin{aligned}e_1 &= 1+e_2\\ e_1+e_2+e_3 &= 0\end{aligned}\right\} \Rightarrow e_3=-1-2e_2 \quad \text{and} \quad \left.\begin{aligned}e_1 &= 1-0.5e_2\\ e_1+e_2+e_3 &= 0\end{aligned}\right\} \Rightarrow e_3=-1-0.5e_2 \tag{2}$$

Symmetry considerations ($e_A = e_i$, $e_B = e_j$, $i, j = 1, 2, 3$, $i \neq j$), valid for isotropic materials, drive to six straight lines forming a hexagon symmetric to the diagonals of the space (e_A, e_B) as shown in Fig. 1.

The hexagon is, simply, the old Tresca failure surface satisfying Eq. (2) in each quadrant. Obviously, an ellipse (e.g. the von Mises one) could be obtained by replacing Eq. (2) by second order

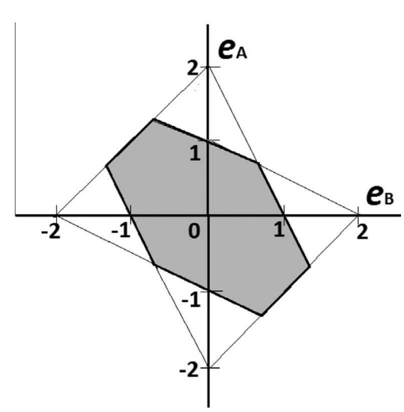

Fig. 1. The Tresca representation of a typical forming limit diagram.