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Mode I fracture initiation in limestone by strain energy density criterion

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ABSTRACT

The critical mode I stress intensity factor obtained from fracture tests on laboratory specimens is often used as fracture toughness in brittle and quasi-brittle materials. However, considerable variations in the experimental results reported for a given material may suggest the dependency of critical mode I stress intensity factor on the geometry and loading conditions of the test specimen. The main purpose of this paper is to study the effect of *T*-stress on the critical mode I stress intensity factor of brittle and quasi-brittle materials. The minimum strain energy density criterion was revisited to take the effect of *T*-stress into account. The results obtained were then compared with a series of experimental results reported for limestone. It is shown as a result that the formulation presented in this paper is capable of estimating the experimental results in a satisfactory manner.

1. Introduction

Fracture toughness is an important property of materials by which the magnitude of the critical far-field load required for the initiation of fracture in a cracked body is determined. The value of fracture toughness is traditionally considered to be the critical mode I stress intensity factor (SIF) which is obtained from fracture test on a pre-cracked laboratory specimen. Classical models of fracture mechanics assume that the critical mode I SIF is not influenced by the geometry and loading conditions. However, many researchers have proven that for small scale yielding, the state of stresses near the crack tip for mode I loading, is affected significantly by the *T*-stress [1–5]. It was found in [1,2] that for the same values of K_I, the sign and magnitude of *T*-stress substantially change the shape and size of the crack tip plastic zone. Subsequently, other researchers [3,4] demonstrated that the near tip stress fields in small scale yielding are affected by a far-field *T*-stress. A negative *T*-stress significantly lowers the crack tip constraint and causes the plastic zone to elongate and rotate forward. Conversely, a positive *T*-stress causes the plastic zone to contract and rotate backward [3,5].

More recently, increasing attention has been devoted to study the effect of higher order terms of Williams' series expansion on the initiation of fracture. By using linear elastic fracture mechanics (LEFMs), several researchers have shown that a fracture model based on the stress intensity factors K_I and K_{II} together with the *T*-stress provides more reliable predictions for the onset of mixed mode brittle fracture [6–9]. Smith et al. suggested a generalized maximum tangential stress (GMTS) criterion and studied the role of *T*-stress in mixed mode brittle fracture [6]. Through their study, they showed that mixed mode brittle fracture is significantly affected by the *T*-stress. Subsequently, Ayatollahi and his co-workers conducted a large number of experiments and found very good agreement between the results predicted by the GMTS criterion and the experimental results [10–13]. However, the GMTS criterion suggests that within the framework of LEFM, the *T*-stress has no influence on mode I fracture resistance of brittle materials.

Nevertheless, in recent studies it has been revealed that there is a significant difference between the experimental results when various test specimens are used to determine the critical mode I SIF for a given brittle material [13–19]. A case in point is the critical stress intensity factor of PMMA reported in [14] which varies approximately from 0.5 to 1.4 MPa $m^{0.5}$. Chao et al. also analyzed a set of experimental results and concluded that the critical SIF of brittle materials is dependent on the specimen geometry and loading conditions [16,17].

Several fracture criteria have been developed to describe brittle failure in linear elastic bodies, when subjected to mixed mode (I/II) loading [20–26]. Among them, the minimum strain energy density (SED) criterion [20,21], proposed by Sih, has been found to be a powerful tool to predict the fracture and fatigue behavior of cracked and notched components [27–33]. In the present study, the effect of *T*-stress on the critical mode I SIF was investigated for brittle and/or quasi-brittle materials. Consequently, the minimum strain energy density (SED) criterion [20,21] has been revisited to take into account the effect of *T*-stress. The results obtained from the SED criterion are then examined using some experimental data reported previously for limestone.

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2. Fracture theory

Elastic stresses around the crack tip can be written as a set of series expansions as

$$\sigma_x = \frac{1}{\sqrt{2\pi r}}\left(K_I\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right)+T+O\left(r^{\frac{1}{2}}\right)$$
$$\sigma_y = \frac{1}{\sqrt{2\pi r}}\left(K_I\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right)+O\left(r^{\frac{1}{2}}\right)$$
$$\tau_{xy} = \frac{1}{\sqrt{2\pi r}}\left(K_I\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)+O\left(r^{\frac{1}{2}}\right) \quad (1)$$

$$\sigma_z = \nu(\sigma_x+\sigma_y) \quad \text{for plane strain}$$

where r and θ are the polar coordinates with the origin located at the crack tip, K_I is the mode I stress intensity factor and υ is the Poisson's ratio. σ_x, σ_y, τ_{xy}, and σ_z are the stresses in the Cartesian coordinates. The first term in each stress component is singular and the second term T, which appears only in σ_x, is constant and independent of the distance r from the crack tip. The higher order terms $O(r^{1/2})$ are often negligible at locations very close to the crack tip.

The SED criterion, formulated by Sih, states that the direction of fracture initiation θ_0 coincides with the direction of minimum strain energy density along a constant radius r_c around the crack tip [20,21]. This can be written mathematically as:

$$\frac{\partial S}{\partial\theta}=0, \quad \frac{\partial^2 S}{\partial\theta^2}>0 \quad (2)$$

The strain energy density factor S can be generally expressed in terms of the stress components as:

$$S = r\frac{dW}{dV} = r\left(\frac{1}{2E}(\sigma_x^2+\sigma_y^2+\sigma_z^2)-\frac{\upsilon}{E}(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x)+\frac{1}{2G}(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right) \quad (3)$$

where dW/dV is the strain energy density, and E and G are the modulus of elasticity and modulus of rigidity, respectively. The SED criterion also suggests that the onset of brittle fracture takes place when the strain energy density factor S along θ_0 and at the critical distance r_c reaches a critical value S_c. Both r_c and S_c are assumed to be constant material properties.

By substituting the stress components for plane strain conditions (Eq. (1)) into Eq. (3) and manipulating the resultant expressions, one may write

$$S=\frac{r(1+\upsilon)}{E}\left(\left(\frac{K_I}{\sqrt{2\pi r}}\right)^2((0.625-\upsilon)+(0.5-\upsilon)\cos(\theta)-0.125\cos(2\theta))+T\left(\frac{K_I}{\sqrt{2\pi r}}\right)\left((0.75-2\upsilon)\cos\left(\frac{\theta}{2}\right)+0.25\cos\left(\frac{5\theta}{2}\right)\right)+0.5T^2(1-\upsilon)\right) \quad (4)$$

If r is replaced by r_c, Eq. (4) can be rewritten as:

$$S\frac{2\pi E}{(1+\upsilon)K_I^2} = ((0.625-\upsilon)+(0.5-\upsilon)\cos(\theta)-0.125\cos(2\theta)) + B\alpha\left((0.75-2\upsilon)\cos\left(\frac{\theta}{2}\right)+0.25\cos\left(\frac{5\theta}{2}\right)\right) + 0.5(1-\upsilon)(B\alpha)^2 \quad (5)$$

The dimensionless parameters B and α in this equation are defined as:

$$B=\frac{T\sqrt{\pi a}}{K_I} \quad (6)$$

$$\alpha=\sqrt{\frac{2r_c}{a}} \quad (7)$$

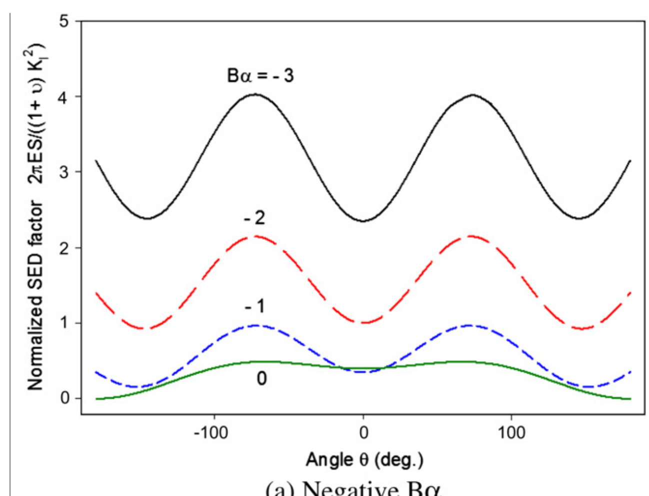
where a is the crack length for edge cracks and the semi-crack length for center cracks, B is the biaxiality ratio, and r_c is the critical distance. The terms involving $B\alpha$ in Eq. (5) represent the contribution of T-stress in the near crack-tip strain energy density.

Fig. 1 shows the variations of normalized strain energy density versus θ (Eq. (5)) for different values of $B\alpha$ when $\upsilon = 0.3$. It is seen from this figure that $B\alpha$ has a considerable influence on the strain energy density. In general, an increase in the absolute value of $B\alpha$ raises the strain energy density too.

By substituting Eq. (5) into Eq. (2), the fracture initiation angle θ_0 is determined by solving

$$((-0.5+\upsilon)\sin(\theta_0)+0.25\sin(2\theta_0))+B\alpha\left((-0.375+\upsilon)\sin\left(\frac{\theta_0}{2}\right)-0.625\sin\left(\frac{5\theta_0}{2}\right)\right)=0 \quad (8)$$

Fig. 2 shows the variations of fracture initiation angle θ_0 versus $B\alpha$ for $\upsilon = 0.3$. It is clear from this figure that when $B\alpha$ is too large (i.e. greater than 0.19), the angle of minimum strain energy density does not coincide with the plane of crack. This point could also be seen in Fig. 1b. Therefore, one may suggest that for high values of T-stress or for large process zone sizes r_c, the crack is likely to kink out of its initial plane. Similar finding has been reported by some other researchers who have studied theoretically and experimentally the phenomenon of crack curving under mode I loading when a large T-stress exists in cracked specimens [6,16,17].



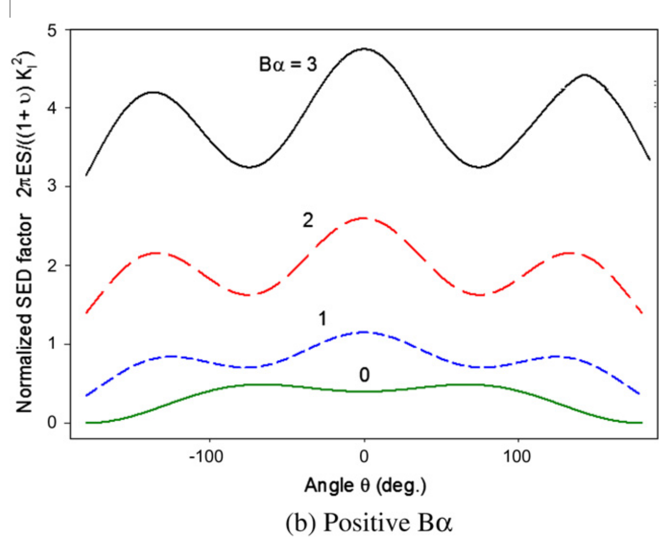


Fig. 1. Variations of the normalized strain energy density versus θ for $\upsilon = 0.3$.