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Discounted cost model for condition-based maintenance optimization

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ABSTRACT

This paper presents methods to evaluate the reliability and optimize the maintenance of engineering systems that are damaged by shocks or transients arriving randomly in time and overall degradation is modeled as a cumulative stochastic point process. The paper presents a conceptually clear and comprehensive derivation of formulas for computing the discounted cost associated with a maintenance policy combining both condition-based and age-based criteria for preventive maintenance. The proposed discounted cost model provides a more realistic basis for optimizing the maintenance policies than those based on the asymptotic, non-discounted cost rate criterion.

1. Introduction

Critical engineering systems in nuclear power plants, such as the reactor fuel core and piping systems, experience degradation due to stresses and unfavorable environment produced by transients or shocks in the reactor. For example, unplanned shutdowns and excursions to poor chemistry conditions result in degradation of components through corrosion, wear and fatigue of material. To control the risk due to failure of critical engineering systems in the plant, maintenance and replacements of degraded components are routinely performed. Because of uncertainty associated with the occurrence of shocks and damage produced by them, theory of stochastic processes plays a key role in estimating reliability and developing cost-effective maintenance strategies.

The failure of a system or structure occurs when its strength drops below a threshold that is necessary for resisting the applied stresses. This paper investigates the reliability of a structure that suffers damage due to shocks arriving randomly in time. Technically the total damage experienced by a system can be modeled as a sum of damage increments produced by individual shocks. To incorporate uncertainties, shocks are modeled as a stochastic point process and the damage produced by each shock is modeled as a positive random variable. In essence, the cumulative damage is modeled as a compound point process [1].

The theory of stochastic processes and its applications to reliability analysis have been discussed in several monographs [2–6]. Mercer [7] developed a stochastic model of wear (degradation) as a cumulative process in which shocks arrive as a Poisson process. A more generalized formulation of the first passage time or reliability function due to damage modeled as a compound renewal process was presented by Morey [8]. Kahle and Wendt [9] modeled shocks as a doubly stochastic Poisson process. Ebrahimi [10] proposed a cumulative damage model based on the Poisson shot noise process. Finkelstein [11] presented a non-homogeneous Poisson process model of shocks and considered the effect of population heterogeneity.

The cumulative damage models are popularly applied to the optimization of maintenance policies using the condition or age based criteria. Nakagawa [12] formulated a preventive maintenance policy, and an age-based policy was analyzed by Boland and Proschan [13]. Later several other policies were investigated by Nakagawa and co-workers [14–16]. Aven [17] presented an efficient method for optimizing the cost rate. An in-depth discussion of inspection and maintenance optimization models is presented in a recent monograph [18]. Grall et al. [19,20] analyzed condition-based maintenance policies by modeling the damage as a gamma process.

Previous studies mostly adopted asymptotic cost rate criterion for optimizing maintenance policies. However, the optimization of discounted cost is more pertinent to practical applications. Our

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experience also suggests that practical applications of compound point processes are limited due to lack of clarity about the mathematical derivations of cost rate and life expectancy. The primary objective of this paper is to present a conceptually clear derivation of discounted cost criterion for optimizing the maintenance of systems subject to stochastic cumulative damage. The proposed derivation is general and it can be reduced to special cases of homogeneous or non-homogeneous Poisson processes, or renewal process.

This paper is organized as follows. In Section 2, stochastic models of degradation and maintenance are presented. Section 3 describes a mathematical framework to evaluate the total expected discounted cost of maintenance. Expressions are derived for three specific maintenance policies. Illustrative examples are presented in Section 4 and conclusions are summarized in Section 5.

2. Stochastic model for degradation

In this paper the degradation is modeled as a stochastic cumulative damage process, where the system suffers damage due to shocks produced by transients (extreme of pressure, temperature and chemical environment). In this model occurrences of shocks are random in time and the damage produced by each shock is also a random variable. The total damage at time t is a sum or cumulation of damage increments produced by all j shocks occurred up to this time. In this section we present some well-known results, see for example Tijms [6] or Nakagawa [18].

Random occurrences of shocks over time, $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_j, \ldots$, are taken as points in a stochastic point process on $[0,\infty)$, as shown in Fig. 1. The total number of shocks in the interval $(0,t]$ is denoted by $\mathcal{N}(t)$ and $\mathcal{N}(0) \equiv 0$.

Define the probability of occurrence of j shocks in $(0,t]$ as

$$H_j(t) = P(\mathcal{N}(t) = j), \tag{1}$$

and the expected number of shocks as

$$R(t) = E(\mathcal{N}(t)). \tag{2}$$

In a given time interval $(0,t]$, the probability associated with the number of shocks (j) is related with that of the time of occurrence of the j th shock ($\mathcal{S}_j$) as

$$F_j(t) = P(\mathcal{S}_j \le t) = P(\mathcal{N}(t) \ge j) = \sum_{i=j}^{\infty} H_i(t). \tag{3}$$

Using this, Eq. (1) can also be rewritten as

$$H_j(t) = P(\mathcal{N}(t) \ge j) - P(\mathcal{N}(t) \ge j+1) = F_j(t) - F_{j+1}(t). \tag{4}$$

Note that $F_j(t)$ depends on the distribution of the time between the shocks.

A shock produces a random amount of damage Y, and its cumulative distribution function is denoted as

$$G(x) = P(Y \le x). \tag{5}$$

The damage occurred at the j th shock is denoted as Y_j.

The evaluation of cumulative damage is based on two key assumptions: (1) damage increments, $Y_1, Y_2, \ldots$, are independent and identically distributed (*iid*), and (2) the damage increments $(Y_j)_{j\ge 1}$ and the shock process $\{\mathcal{N}(t) : t \ge 0\}$ are independent.

The total damage caused by j shocks is given as

$$D_j = \sum_{i=1}^{j} Y_j, \quad j \ge 1, \tag{6}$$

and $D_0 \equiv 0$. The distribution of D_j is obtained from the convolution of $G(x)$ as

$$P(D_j \le x) = G^{(j)}(x), \tag{7}$$

where

$$G^{(j+1)}(x) = \int_0^x G^{(j)}(x-y)\,\mathrm{d}G(y) = \int_0^x G(x-y)\,\mathrm{d}G^{(j)}(y). \tag{8}$$

Note that $G^{(0)}(x) = 1,\ x \ge 0$. The total damage, $Z(t)$, at time t, however, depends on the number of shocks $\mathcal{N}(t)$ occurred in this interval, i.e.,

$$Z(t) = \sum_{j=1}^{\mathcal{N}(t)} Y_i = D_{\mathcal{N}(t)}. \tag{9}$$

Using the total probability theorem and independence between the sequence $Y_1, Y_2, \ldots$ and $\mathcal{N}(t)$, we can write for $x > 0$

$$P(Z(t) > x) = \sum_{j=1}^{\infty} P(D_j > x, \mathcal{N}(t) = j) = \sum_{j=1}^{\infty} (1 - G^{(j)}(x)) H_j(t). \tag{10}$$

Using the facts that $\sum_{j=0}^{\infty} H_j(t) = 1$ and $G^{(0)}(x) = 1$, the distribution of the total damage can be written as

$$P(Z(t) \le x) = H_0(t) + \sum_{j=1}^{\infty} G^{(j)}(x) H_j(t) = \sum_{j=0}^{\infty} G^{(j)}(x) H_j(t). \tag{11}$$

Substituting $H_0(t) = 1 - F_1(t)$ and $H_j(t) = F_j(t) - F_{j+1}(t)$ from Eq. (4), it can be written as

$$P(Z(t) \le x) = 1 - \sum_{j=1}^{\infty} [G^{(j-1)}(x) - G^{(j)}(x)] F_j(t). \tag{12}$$

This is a fundamental expression that can be used to compute the system reliability. Suppose damage exceeding a limit z_F causes the component failure, Eq. (12) provides $P(Z(t) \le z_F)$ which is synonymous with the reliability function.

We conclude this section with a formula for the mean value of the first time τ_B that the total damage exceeds a level B

$$\tau_B = \min\{t : Z(t) > B\}. \tag{13}$$

So $\{\tau_B > t\} = \{Z(t) \le B\}$ and

$$E(\tau_B) = \sum_{j=0}^{\infty} G^{(j)}(B) \int_0^{\infty} H_j(t)\,\mathrm{d}t. \tag{14}$$

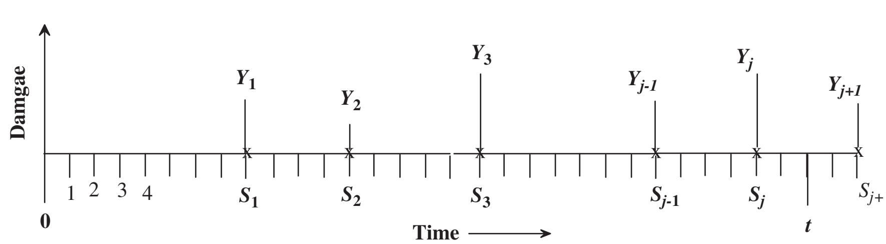

Fig. 1. A schematic of the stochastic shock process causing random damage.