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Modelling uncertainties in the diffusion-advection equation for radon transport in soil using interval arithmetic

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ABSTRACT

Modelling radon transport in the earth crust is a useful tool to investigate the changes in the geo-physical processes prior to earthquake event. Radon transport is modeled generally through the deterministic advection-diffusion equation. However, in order to determine the magnitudes of parameters governing these processes from experimental measurements, it is necessary to investigate the role of uncertainties in these parameters. Present paper investigates this aspect by combining the concept of interval uncertainties in transport parameters such as soil diffusivity, advection velocity etc, occurring in the radon transport equation as applied to soil matrix. The predictions made with interval arithmetic have been compared and discussed with the results of classical deterministic model. The practical applicability of the model is demonstrated through a case study involving radon flux measurements at the soil surface with an accumulator deployed in steady-state mode. It is possible to detect the presence of very low levels of advection processes by applying uncertainty bounds on the variations in the observed concentration data in the accumulator. The results are further discussed.

1. Introduction

Radon (^{222}Rn) is a naturally occurring radioactive noble gas emanating from all rocks and soils due to the radioactive decay of radium atom naturally present inside the material grains. It has been used as a trace gas in terrestrial, hydrogeological and atmospheric studies, because of its ability to travel to comparatively long distances from host rocks as well as its efficient detection at very low levels. Radon, as a potential earthquake precursor, has been monitored for decades along with several other precursors, in the anticipation that any seismic or tectonic activity will lead to stresses responsible for the anomalous release of these precursors. However, the predictability of an earthquake based on these precursors is still elusive. This is mainly because of uncertainties in the observed values of the precursors, arising out of interferences due to environmental parameters. In case of ^{222}Rn gas, the most important interfering parameters are temperature, humidity and pressure differences between soil pore and atmosphere, which lead to diurnal and seasonal variations in the concentrations of the gas. The porosity, grain size, and flow velocity of the driving gas are equally significant in affecting the monitored concentration of ^{222}Rn gas. In view of this, the correct interpretation of real-time monitored data of ^{222}Rn and its correlation with seismic events requires better understanding of the radon transport process in soil pore and its perturbation during earthquake events.

Radon transport in soil pore is mainly governed by two physical processes namely, diffusion and advection. Different models have been developed based on these transport processes to study the anomalous behaviour of soil radon in geothermal fields, thermal spas, active faults, volcanic processes and seismotectonic environments. These models have been employed to estimate process driven parameters (such as diffusion coefficient, carrier gas velocity) from the measured data of soil radon. Estimates of these parameters may deviate significantly from the true values, if the uncertainty associated with the various input parameters of the models are not taken into consideration.

The radon transport models employ diffusion and advection governing differential equations, ordinary or partial, that can be investigated by analytical or numerical methods (Ames, 2014; Langtangen, 2013). The solution of the steady state radon transport equations, and several parameters involved in these equations have been extensively investigated. These studies include, diffusion and exhalation of ^{222}Rn and ^{220}Rn from building materials (Folkerts et al., 1984; Fournier et al., 2005), radon exhalation in an Indian Uranium tailings pile (Sahoo et al., 2010), radon exhalation rates from concrete surfaces (Yu et al., 1996; Renken and Rosenberg, 1995) effects of cracks

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and holes on the exhalation of radon from concrete (Man and Yeung, 1997 Dimbylow and Wilkinson, 1985), two dimensional diffusion theory of trace gas build-up in soil chambers for flux measurements (Sahoo and Mayya, 2010). Further, various investigations such as radon generation and transport in porous medium (Rogers and Nielson, 1991), flow and diffusion of radon isotopes in fractured porous media (Schery et al., 1988), modelling radon transport in dry, cracked soil (Holford et al., 1993) and modelling radium and radon transport through soil and vegetation (Kozak et al., 2003) have also been attempted. Procedures for the determination of ^{222}Rn exhalation and effective ^{226}Ra activity in soil samples were given by Escobar et al. (1999), while the effect of soil parameters on radon entry into a building by means of the *transrad* numerical model has been proposed by Albarracin et al. (2002). Jiranek and Svoboda (2009) modeled a new technique for more precise determination of the radon diffusion coefficient. The Radon diffusion through soil and into air investigated (Savovic et al., 2011) and the solution of the relevant diffusion equation is given using the explicit finite difference method. Transient diffusion measurement of radon in Japanese soils from a mathematical viewpoint discribed by Sasaki et al. (2006). Varhegyi et al. (1992) briefly outlined the theory of radon transport by geogas microbubbles.

The variables and parameters appearing in the radon transport differential equations are generally considered as crisp in most formulations. However, the inherent errors in observations and measurement methodologies may introduce uncertainty in these parameters, transforming them to intervals or fuzzy numbers instead of crisp values. Several methods have been developed to handle differential equations involving uncertain or fuzzy parameters. For example (Kaleva, 1987), discussed fuzzy differential equations. Numerical solution of fuzzy differential equations of nth order using Runge Kutta method was given by Parandin (2012). A new analytical method for solving n-th order fuzzy and interval differential equations using center based approach was proposed by Tapaswini and Chakraverty (2014a,b,c). Solutions of fuzzy differential equations based on generalized differentiability have been investigated by Bede and Gal (2010). Nayak and Chakraverty (2015a,b) proposed numerical solution of uncertain neutron diffusion equation and used fuzzy finite element analysis for solving it. Non-probabilistic uncertainty analysis of forest fire model by solving fuzzy hyperbolic reaction-diffusion equation was investigated by Tapaswini and Chakraverty (2014a,b,c).

This paper gives a brief background of the radon transport with uncertainty. The analytical approach for solving the diffusion equation using “crisp” and “parameter concept” using “Interval Arithmetic” is described. Numerical examples have been investigated and finally, conclusions drawn from the examples are presented.

2. Interval arithmetic and parametric approach

Well-known interval arithmetic has been incorporated in the Appendix. As such, parametric approach is used here to represent an interval in crisp form. In this approach, the interval $\widetilde{X} = [\underline{X}, \overline{X}]$ may be written as (Behera and Chakraverty, 2015; Tapaswini and Chakraverty, 2014a,b,c)

$$X = \beta(\overline{X} - \underline{X}) + \underline{X}$$

where $0 \le \beta \le 1$. It can also be written as

$$X = 2\,\beta\, \Delta\widetilde{X} + \underline{X}$$

The lower and upper bounds of the solution can then be obtained by substituting $\beta = 0$ and 1 respectively as follows:

$$\widetilde{X} = \underline{X} \text{ when } \beta = 0,$$

$$\widetilde{X} = \overline{X} \text{ when } \beta = 1.$$

3. Radon transport formulation - crisp approach

3.1. Diffusive transport

Consider the case of radon transport, due to diffusion alone, occurring in vertical direction i.e. x-direction after emanating from soil grain to pores space and let $C(x)$ is the steady state concentration in the soil pore space. If soil properties and radioactivity distributions are assumed to be homogenous, then the profile $C(x)$ satisfies the following continuity equation

$$D\frac{\partial^2 C(x)}{\partial x^2} - \lambda C(x) + \lambda C_\infty = 0 \tag{3.1}$$

The first term and the second term of Eq. (3.1) represent the loss of radon in the pore space of the soil by diffusion and radioactive decay respectively, while the third term represents the production of radon due to emanation from soil grain to pore volume.

Here, λ: radioactive decay constant for radon = 2.1×10^{-6} (s^{-1}) and

$$C_\infty = \frac{R\rho E}{n} \tag{3.2}$$

Where, R is the ^{226}Ra content (Bq kg^{-1}) in soil, D is the diffusion coefficient of radon in the soil ($m^2 s^{-1}$), ρ is the dry bulk density (kg m^{-3}) of soil, n is the porosity of the soil, and E is the radon emanation factor.

The boundary conditions of the problem may be written as

$$C(x = -\infty) = C_\infty \tag{3.3}$$

$$C(x = 0) = C_0 \tag{3.4}$$

An analytical steady state solution of Eq. (3.1) is given by

$$C(x) = C_\infty + C_1 e^{\left(\frac{x}{L}\right)} + C_2 e^{\left(-\frac{x}{L}\right)} \tag{3.5}$$

Where, $L = \sqrt{\frac{D}{\lambda}}$, is the radon diffusion length in soil.

By using boundary conditions, the exact solution of the steady state diffusion Eq. (3.1) may be written as

$$C(x) = C_0 + (C_0 - C_\infty)e^{\frac{x}{L}} \tag{3.6}$$

The radon surface flux density (F_S) due to diffusion may be obtained from Eq. (3.6) as:

$$F_S = -D\frac{\partial C(x)}{\partial x}(x = 0) = \frac{D(C_\infty - C_0)}{L} \tag{3.7}$$

3.2. Diffusio-advective transport

In addition to diffusion, advection may transfer radon over a wide range of distances, depending on the porosity and velocity of the carrier fluid. Due to this reason, soil gas radon has been established as a powerful tracer used for a variety of purposes, such as exploring uranium ores, locating geothermal resources and hydrocarbon deposits. Eq. (3.1) may be modified to include the advection process as follows:

$$D\frac{\partial^2 C(x)}{\partial x^2} - u\frac{\partial C(x)}{\partial x} - \lambda C(x) + \lambda C_\infty = 0 \tag{3.8}$$

where, u=velocity of carrier gas (m s^{-1})

It may be noted that the boundary conditions remain unaltered and are given by Eqs. (3.3) and (3.4).

An analytical steady state solution of Eq (3.8) is given as

$$C(x) = C_\infty + C_1 e^{\left(\frac{u}{2D} + \sqrt{\frac{u^2}{(2D)^2} + \frac{\lambda}{D}}\right)x} + C_2 e^{\left(\frac{u}{2D} - \sqrt{\frac{u^2}{(2D)^2} + \frac{\lambda}{D}}\right)x} \tag{3.9}$$

Using the boundary conditions, the exact solution of the advection inclusive Eq (3.8) may be written as