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Application of fractional-derivative standard linear solid model to impact response of human frontal bone

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ABSTRACT

The impact of a rigid body on a thin plate with a buffer is investigated in this paper. A buffer is assumed as a linear spring fractional derivative dashpot which exhibits the viscoelastic features. The fractional-derivative standard linear solid model is suggested for describing the shock interaction of the impactor with a circular elastic plate. We assume that a transient wave of transverse shear is generated in the plate and the reflected wave does not have sufficient time to interact with the plate before the impact process is completed. The ray method is used outside the contact spot, but the Laplace transform method is applied within the contact region. The time-dependence of the contact force is determined. A numerical example is carried out by considering crash scenarios in frontal impacts of the human head which could estimate brain injury risks.

1. Introduction

There are a number of situations in which the human body is subjected to impact loading, including automobile crashes, falls, blast effects, and high-energy sports. In the present paper, the human head response to drop mass impacts against frontal bone will be considered which is important for the analysis of human brain injury of sportsmen or automobile accident occupants. To determine the impact force that is actually being transmitted to bone will require analytical formulation and numerical calculations.

Although bone has been known to behave viscoelastically [1,2]. Detailed experimental works on the viscoelasticity of bone have been only carried out recently. It was reported in [3] that the standard linear solid model gave a good fit to experimental data for human cranial bone. The simplest models of linear viscoelasticity used for describing viscoelastic properties of bone have been discussed in [4].

It should be emphasized that in recent decades *fractional calculus* (integral and differential operators of noninteger order) has been the object of ever increasing interest in many branches of natural science, and of engineering interest as well. The application of fractional calculus to dynamic problems of linear and nonlinear hereditary mechanics of solids was reviewed in [5]. Discussed in [6] is the employment of fractional calculus techniques to problems in biophysics, while Magin [7] has surveyed the examples of its usage in bioengineering. In was noted that fractional calculus has attracted limited attention in the field of biomechanics, and then has underlined that "this is surprising because the methods of fractional calculus, when defined as a Laplace or Fourier convolution product, are suitable for solving many problems in biomedical research".

In recent years, this situation has been changes drastically. The linear viscoelastic models based on the operators of the fractional order have found a wide use in different problems of bioengineering, among them, for describing viscoelastic features of bones and soft tissues. Using the experimental data [8] of human cranial bone, it was shown [3] that constitutive equations of viscoelastic materials involving fractional derivatives of different orders could be used with a success for describing viscoelastic features of bone. Suggested in [9] is the modification of the Biot theory using fractional calculus to describe interactions between fluid and solid structure in cancellous bone. Experimental validation of this model using samples of human cancellous bone produced excellent agreement between theory and experiment on ultrasonic wave propagation through bone samples.

Since the experimental results [3] showed that human skull structure belongs to small damped structure, we will utilize the fractional derivative standard linear solid model to describe the impact response of human frontal bone under high-energy impact conditions, such as motor vehicle collisions or shock interactions of sportsmen on sports fields. Adopt the scheme suggested in [10] and generalize their fractional-derivative viscoelastic model of the shock interaction of a rigid body with a plate for the case of impact response of human frontal bone.

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2. Problem formulation

Consider a situation when human head is attacked by a rigid body in the region of its forehead. It is necessary to understand how much force is being transferred into the load-bearing bones of the body during such a collision. Here consider the model of a human frontal bone as a circular isotropic plate, while viscoelastic properties of soft tissues in the place of contact are simulated by a fractional derivative standard linear solid model.

Thus, let a rigid cylindrical body of mass m and radius r_0 with the initial velocity V_0 impact an Uflyand–Mindlin plate of infinite extent (this assumption is introduced due to the short duration of contact interaction in order to ignore reflected waves) with thickness h. The role of soft tissues between the impactor and the bone will be played by a buffer, viscoelastic features of which are described by a fractional derivative standard linear solid model (Fig. 1). At the moment of impact, shock waves are generated in the plate, which then propagate along the plate with the velocities of transient elastic waves. Moreover we shall assume during the impact process transverse forces and shear deformations predominate in the plate's stress-deformed state in vicinity of the contact spot (the contact region of plate and buffer interaction).

The dynamic behavior of an Uflyand–Mindlin plate behind the transient elastic wavefronts is described in the polar coordinates by the following equations [10]:

$$\frac{\partial Q_r}{\partial r} + \frac{1}{r} Q_r = \rho h \dot{W}, \tag{1a}$$

$$\dot{Q}_r = K\mu h\left(\frac{\partial W}{\partial r} - B_r\right), \tag{1b}$$

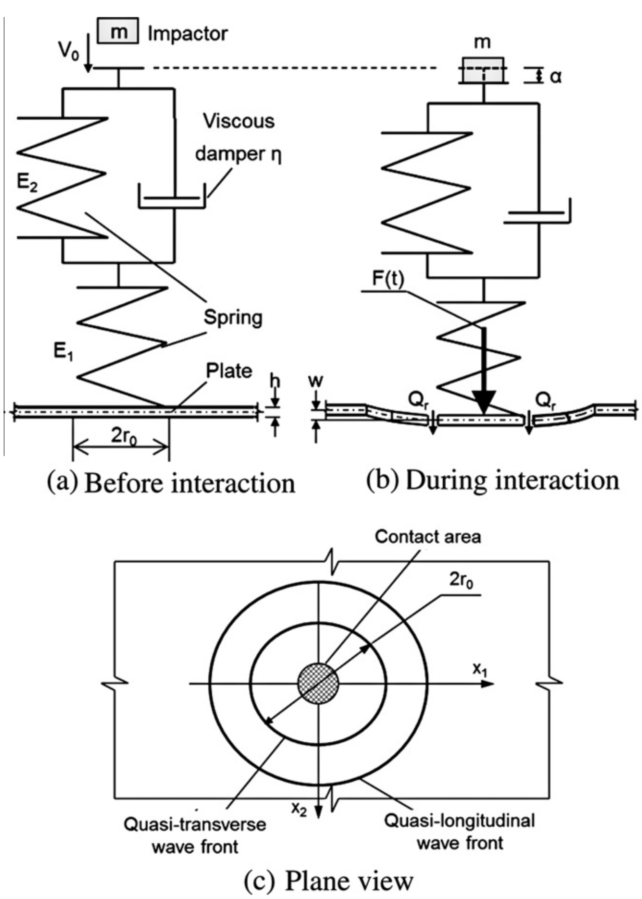

Fig. 1. Scheme of the shock interaction of a rigid body and a buffer embedded in an Uflyand–Mindlin plate.

$$\frac{1}{r}(M_r - M_\varphi) + \frac{\partial M_r}{\partial r} + Q_r = \frac{\rho h^3}{12}\dot{B}_r, \tag{2a}$$

$$\dot{M}_r = D = \left(\frac{\partial B_r}{\partial r} + \sigma\frac{B_r}{r}\right), \tag{2b}$$

$$\dot{M}_\varphi = D\left(\frac{B_r}{r} + \sigma\frac{\partial B_r}{\partial r}\right), \tag{2c}$$

where r and φ are the polar radius and angle, respectively, M_r and M_φ are the bending moments, Q_r is the shear force, B_r is the angular velocity of rotation of the normal to the plate's middle surface in the r-direction, $W = \dot{w}$ is the deflections velocity, D is the cylindrical rigidity, ρ is the density, K is the shear coefficient, μ is the shear modulus, σ is Poisson's ratio, and an over dot denotes the time derivative.

The equations of motion of the present impact problem are given by (Fig. 1)

$$M(\ddot{\alpha} + \ddot{w}) = -F, \tag{3}$$

$$\rho h \pi r_0^2 \ddot{w} = -2\pi r_0 Q_r|_{r=r_0} + F, \tag{4}$$

subjected to the initial conditions

$$\alpha|_{t=0} = w|_{t=0} = \dot{w}|_{t=0} = 0, \quad \dot{\alpha}|_{t=0} = V_0, \tag{5}$$

where α and w are the displacements of the upper and lower points of the buffer, respectively.

The contact force F is connected with the difference in displacements of the buffer's upper and lower ends by the generalized standard linear solid model with the Riemann–Liouville derivative as

$$(1 = \tau_\varepsilon^\gamma D^\gamma)F = E_0(1 = \tau_\sigma^\gamma D^\gamma)(\alpha - w), \tag{6}$$

$$E_0 = \frac{E_1 E_2}{E_1 + E_2}, \quad \tau_\varepsilon = \frac{\eta}{E_1 + E_2}, \quad \tau_\sigma = \frac{\eta}{E_2}, \tag{7}$$

where τ_e and τ_σ are the relaxation and retardation times, respectively, E_0 is the relaxed elastic modulus, and $\gamma (0 < \gamma \leqslant 1)$ is the order of the fractional derivative (fractional parameter) defined by

$$D^\gamma F = \frac{d}{dt}\int_0^t \frac{F(t - t')}{\Gamma(1-\gamma)t'^\gamma}dt'. \tag{8}$$

3. Method of solution

The methods utilized for solving the given problem within and out of the contact region are different, namely, the ray method is used outside the contact spot, but the Laplace transform method is applied within the contact region.

3.1. The ray method

Consider a shock wave in the plate as a layer of thickness δ, within which the desired function changes from the magnitude Z^- to the magnitude Z^+, but remaining a continuous function. Then integrating Eqs. (1) and (2) over the layer's thickness from $-\delta/2$ to $\delta/2$, with tending δ to zero, and considering that inside the layer the condition of compatibility [11] is fulfilled in the form of

$$\dot{Z} = -G\frac{\partial Z}{\partial r} + \frac{\delta\sigma}{\delta t}, \tag{9}$$

where G is the normal velocity of wave surface, and $\delta/\delta t$ is the δ-derivative with respect to time, we find the dynamic conditions of compatibility

$$[Q_r] = -\rho G h[W], \ -G[Q_r] = K\mu h[W], \tag{10}$$

$$[M_r] = -\frac{\rho h^3}{12}G[B_r], \ -G[M_r] = D[B_r] \tag{11}$$

where $[Z] = Z^+ - Z^-$.