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# Uncertainties of unidirectional composite strength under tensile loading and variation of environmental condition

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ABSTRACT

A probabilistic strength model is developed for unidirectional composites with fibers in hexagonal arrays. The model assumes that, a central core of broken fibers surrounded by unbroken fibers which are subjected to unidirectional tensile loading. The proposed approach consists in using a modified shear lag model to calculate the ineffective lengths and stress concentrations around fiber breaks. The main feature in the model lies in incorporating the variation of composite properties due to temperature and moisture, in order to predict degradation of fibers and matrix characteristics. The strength degradation is often seen as a result of changes in ineffective lengths at fiber breaks, leading to stress concentrations in intact neighboring fibers. As fiber breaks are intrinsically random, the variability of input data allows us to describe the probabilistic model by using the Monte-Carlo method. The sensitivities of the mechanical response are evaluated regarding the uncertainties in design variables such as Young's modulus of fibers and matrix, fiber reference strength, shear yield stress, fiber volume fraction and shear parameter defining the shear stress in the inelastic region.



## 1. Introduction

Carbon fibers reinforced polymers are widely used for mechanical and structural components because of their high specific strength and rigidity, and excellent durability. Because such reinforcing fibers are generally brittle and have a large scatter in strength, the mechanical properties of composites have often been discussed from the viewpoint of reliability engineering. Predicting the strength of these materials from the properties of their constituents is a task which has not been yet solved for all classes of materials. From the literature, the oldest models for predicting the strength of polymer matrix composites [1,2] are related the failure of the fiber bundles in the manufacturing of the matrix materials. These models used the ineffective length to estimate the tensile strength, on the basis of shear-lag analysis. However, they did not consider the effects of stress concentrations in intact neighboring fibers.

Statistical strength and rupture lifetimes of unidirectional model carbon fiber/epoxy matrix micro-composites are given for seven parallel carbon fibers forming approximate hexagonal packing embedded in an epoxy matrix [3]. Using the finite elements to model the matrix and continuous one-dimensional springs to model the fibers, the question is addressed how to choose the effective dimensions of the matrix springs connecting the neighboring fibers [4]. Their model also considered direct interactions between the broken fibers and the nearest neighbors. Later, this model is extended [5] to account for the effects of interface sliding, axial matrix stiffness and uneven fiber positioning, on stress concentrations in intact neighboring fibers. The peak stress concentration in intact fibers near the broken fiber occurred slightly out of the rupture plane is also showed [6,7]. An entirely different analysis model is proposed to study the stress field in a general unidirectional composite material containing fiber fractures [8]. This model is based on approximating annular ring of fibers to represent the unbroken neighboring fibers. Multiple fiber breaks were modeled by a fiber discount methodology. The problem of a penny shaped crack in the center of multiple concentric cylinders is also considered in many papers [9]. This problem was solved by applying standard elasticity assumptions, with appropriate choice of stress functions in each constituent. Using geometrical considerations, this solution was applied to the problem of a fiber fracture in unidirectional composite materials. To predict the strength of fiber reinforced composites under tensile and bending loading of beams, a direct numerical simulations and analytical models are proposed [10].

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Using a regular square array of intact neighboring fibers, it determined the strength of a Ti-6Al-4V matrix reinforced with Sigma SiC fibers. It was shown that random fiber fractures occur at loads below the ultimate composite strength level, and the statistical accumulation of these flaws led to composite failure. Using Batdorf probabilistic framework for tensile strength, the load-sharing analysis is introduced in order to compute the shear stress distribution [11]. This model is based on shear-lag assumptions with geometric assumption to smear the cluster of fiber fractures. They incorporated the shear yielding at the fiber/matrix interface by defining a shear parameter which is used to define the complete fiber/matrix debonding in elastic perfectly plastic matrix behavior. A probabilistic strength model based on Markov process for unidirectional composites with fibers in hexagonal arrays is developed to have the stress concentration distribution and rupture fibers probability [12]. The model assumes that a group of fiber breaking points, a so-called cluster, evolves with increased stress. The cluster evolution process branches because of various fiber-breakage paths. Based on the fragmentation analysis of the fibers, a progressive damage model for fiber reinforced composites is presented in [13]. The stiffness loss of a unidirectional composite comes from the Weibull distribution of fiber strength and the mechanical properties of the fiber, the matrix and the interface.

Numerical micromechanical investigations of the damage evolution are carried out for unidirectional glass fiber reinforced composites [14]. It demonstrated that fibers with constant strength ensure higher strength of a composite at the pre-critical load, while the fibers with randomly distributed strengths lead to higher strength of the composite at post-critical loads. Using the basic equations for axi-symmetrical elasticity and the boundary conditions to ensure continuity, periodicity, and surface conditions of matrix and fibers, an analytical model to estimate the average stresses and the stress intensity factor for multiple cracking of fiber/matrix composites is proposed [15].

The design of a composite using fiber and matrix properties requires micro-mechanical modeling of strength. Therefore, the composite material strength has an inherent variability which is an image of the underlying failure processes of fibers and matrix under varying environmental conditions, such as temperature and moisture concentration. For fiber dominated tensile composite failures, the failure process starts with the breaking of weak fibers. The initial failure sites are isolated by micro-redundancy provided by local load-sharing of the neighboring fibers. Higher applied load leads to progressive clustering of fiber breaks which ultimately cause macroscopic failure. Probability modeling of the failure sequences of the fiber breaks and the spatial geometry of the clustering of the sites lead to the local load-sharing model.

The simulation by the Monte Carlo method has been recognized as a powerful tool for modeling the strength distribution of random systems, such as unidirectional composites under tensile load. It allows us to determine statistical fiber strength, with few simplifying assumptions related to approximate analytical techniques. In this scope, a simplified stochastic two-dimensional model to predict the strength distribution of single-ply unidirectional composites is proposed [16]. The model is studied through Monte Carlo simulations and accounts for the following parameters: specimen size, fiber strength distribution, fiber–matrix properties and load transfer at broken fibers. In the study, Monte Carlo simulations using 2-D and 3-D (square and hexagonal array), was carried out on unidirectional graphite/epoxy and glass/polyester composites [17]. The results obtained by using 3-D hexagonal array model show a good agreement with the experimental data describing the tensile strength and the failure process of unidirectional composites. Using the study of the heat generation mechanism in unidirectional composites under tensile impact and of the interaction between the temperature and the composite mechanical behavior, a dynamic Monte Carlo microscopic damage constitutive model taking into account the thermo-mechanical coupling is proposed [18].

In the present paper, the main strength prediction model is based on method proposed in reference [11]. In order to predict the unidirectional composite strength degradation, the variation of mechanical characteristics is incorporated in this model. The strength degradation is a result of the changes in ineffective lengths at fiber breaks and the corresponding stress concentrations in intact neighboring fibers. After the calculation of stress concentrations and ineffective lengths, the Monte-Carlo method is used to predict the cumulative distribution function of the strength. The sensitivities to mechanical properties, applied loading, environmental conditions, and volumetric fraction uncertainties are evaluated and conclusions are drawn concerning the relative importance of these variables in unidirectional composite design problems.

## 2. Mechanical model

### 2.1. Micromechanical characterization

Hygrothermal behavior of carbon/polymer laminates is mainly determined by the matrix and the interface properties since carbon fibers are relatively insensitive to moisture and temperature elevation. Moisture effect generates the residual hygral stresses, interfacial degradation, and polymer plasticization [19–21]. Moreover, the plasticization can result in decreased glass transition temperature [22] which may affect the composite behavior in elevated temperature environments. To introduce the effect of temperature and moisture concentration variation on the mechanical properties is proposed [22], the non-dimensional temperature $T^*$, which is the main parameter for evaluating the composite hygrothermal characteristics

$$T^* = \frac{T_g - T_{opr}}{T_g - T_{rm}} \tag{1}$$

where $T_g$ is the glass-transition temperature, $T_{opr}$ is the operation temperature and $T_{rm}$ is the room temperature. It is also assumed that the moisture suppresses the glass transition temperature $T_g^0$ by a simple moisture shift $g$ as

$$T_g = T_g^0 - gC \tag{2}$$

where C is the moisture content.

The exponents of $T^*$ are used to empirically fit the matrix and fiber stiffness $(E_m, E_f)$ and strength $(X, X', Y, Y')$ data in terms of moisture and temperature:

$$E_m = E_m^0 (T^*)^a \tag{3}$$

$$\frac{E_f}{E_f^0} = \frac{G_f}{G_f^0} = (T^*)^f \tag{4}$$

where the superscript '0' indicates the initial values of matrix and fiber stiffness and strengths. For our case, the hygrothermal parameters and the power variables $a$ and $f$ are given in Table 1.

**Table 1**
Temperature and moisture parameters [22].

| $T_g^0$ (°C) | $T_{rm}$ (°C) | $g$ (°C/C) | $a$ | $f$ |
|---|---|---|---|---|
| 160 | 22 | 2000 | 0.5 | 0.04 |