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Energy density mechanics applied to coating blistering problems

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ABSTRACT

The use of fracture mechanics outside the area of mechanics, including materials science, is evidence that the concept is capable of handling failures involving almost any types of crack propagations. This paper outlines the simple method on how to use fracture mechanics concepts in coating study by using a concrete example that is easy to follow. The example presented here is the use of fracture mechanics to develop equation governing the blister propagation using the concept pioneered by Sih, the SED concept. Despite being simple, the method gives excellently good agreement with those available solutions derived by other methods.

1. Introduction

From mechanics point of view, adhesion failure is similar to that of fracture [1]. It may be divided into two general classifications: (i) cohesive fracture, or the separation of a material from itself, and (ii) adhesive fracture, or the separation of a material from a dissimilar material at the bond line between the two materials. In both cases, the concept of fracture mechanics can be useful.

Meanwhile, most of governing equations for blistering are derived directly based on the energy release rate method. This limits the experimental verification due to equipment restrictions in most of the small and medium laboratories. This research aims to bring the fracture mechanics concept, specifically utilizing the strain energy density (SED) concept, into coating study to enrich the available equations that are usually developed based upon energy release method. The SED concept usage in fracture mechanics is pioneered by Sih, who showed that for a combined mode of loading, it is suitable to use the SED approach to describe crack behavior [2]. He further showed that conditions for crack propagation in a material could be estimated using the strain energy density factor S that can be related to stress intensity factor (SIF). For modes I and II mixed mode the SED coefficient becomes:

$$S = a_{11}K_I + 2a_{12}K_IK_{II} + a_{22}K_{II}, \tag{1}$$

where S is SED coefficient; whereas K is stress intensity factor (SIF), and a_{11}, a_{12}, and a_{22} are unique constants dependent upon Poisson's ratio and Young's modulus [2]. One of the benefits of such attempt is the easiness to verify experimentally in ordinary laboratories and simple computational approaches due to the simple use of SIF.

Even if nowadays the surface engineering has developed to beyond coating, the blistering remains one of the most serious problems. Understanding the physical and metallurgical phenomena of blistering remains important. In this research, blister development in coating degradation is not discussed in depth. The readers are advised to find it in any coating textbooks e.g., [3]. Being a part of larger research, this paper only discusses about the development of governing equation for blister development that is easy to verify experimentally. The result will then be combined with other corrosion and diffusion theories to form a unified equation. The adhesion type here is limited to suctioning, similar to that described in the standard method [4].

2. Review of related works

The first model to use mechanics concept was developed as early as 1995 [5]. Since then, the method has become popular in this area of research (e.g., [6,7]). It was formulated by using the energy balance. The total energy U_T of a blistered body can be written as

$$U_T = U_T + U_E + U_S, \tag{2}$$

where U_P is potential energy of externally applied load, U_E is elastic energy stored in the system, and U_S is surface energy. If the fresh area created by blister is dA, the three different conditions can be thought here.

Case 1. Crack retreat, dA is negative.
Case 2. Equilibrium, no crack propagation, dA is zero.
Case 3. Crack propagate, dA is positive.

Meanwhile, the potential energy is

$$U_P = -\int pdV|_p = -pV = -C_1\pi a^2 y_c p, \tag{3}$$

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where p is net pressure, a is blister radius, and y_c is deflection (blister height).

The elastic energy is

$$U_E = -\int p(y_c)dV|_p = \frac{pV}{4} = \frac{C_1\pi a^2 y_c p}{4}. \quad (4)$$

And eventually, by assuming that the condition resembles those of plain stress and simplified into mode I problem, the stress intensity factor becomes:

$$K_I = \sqrt{\frac{5C_1}{4}E \cdot y_c \cdot p}, \quad (5)$$

where the value of C_1 is fitted with experimental results.

Some used energy method based on fracture mechanics parameters effective normal (N) and shear (S) membrane stresses and the bending moment (M) [6]. Expressed in SIF, their equation becomes:

$$K_I = \sqrt{\frac{6(1-\nu^2)}{h^3}\left(M^2 + \frac{h^2N^2}{12}\right) + \frac{S^2(1+\nu^2)}{h}}, \quad (6)$$

where ν is the Poisson's ratio of the coating, and h is the coating thickness. Eq. (6) can be used with extensive knowledge of the coating conditions, including normal (N) and shear (S) membrane stresses and the bending moment (M). Other available mathematical models are shown in Table 1.

3. Fracture mechanics approach

3.1. Fracture mechanics and weight function approach to blister

Fracture mechanics applications on areas other than mechanics are enormous, especially in materials science area. The common use of it is to calculate the driving force on a crack and to characterize the material's resistance to fracture using analytical solid mechanics and experimental solid mechanics.

Fig. 1 shows the very basic typical usage of fracture mechanics. In the figure, only mode I is shown. The corresponding relation between the stresses at locations to the stress singularity coefficient for mode I is shown as:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{3}\sin\frac{3\theta}{2}\right), \quad (7a)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right), \quad (7b)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}. \quad (7c)$$

Table 1
Summary of the model compared with other available models.

Model	Developed by
$\frac{\alpha E\sqrt{y_c}}{(1-\nu^2)}(a_{11}\cdot p_x + 2a_{12}\cdot p_x\cdot p_y + a_{22}\cdot p_y)$	Prawoto (current research)
$\frac{p\cdot\sqrt{(1-\nu^2)\cdot t}}{2}$ where t is the coating thickness	Bresser et al. [10]
$\sqrt{p\cdot C\cdot y_c\cdot E}$ where C is the constant dependent upon geometry	Galindo et al. [13]
$\frac{p\sqrt{(1-\nu^2)}}{2y_c}$	Jahnsen [6]
$(p^4r^4E^5/17.4\cdot t)^{1/6}$ where t is the coating thickness	Kappes et al. [12]
$\frac{p\cdot y_c\sqrt{(1-\nu^2)}}{2f}$ where f is the constant dependent upon geometry	Volinsky et al. [7]
$\frac{E}{2}\sqrt{5\cdot C\cdot p\cdot y_c}$ where C is the constant dependent upon geometry	Wan and Mai [5]

In alphabetical order. See Fig. 7 for graphical depiction general description: p blister pressure, y_c blister height, t coating thickness, E Young's modulus, r blister radius.

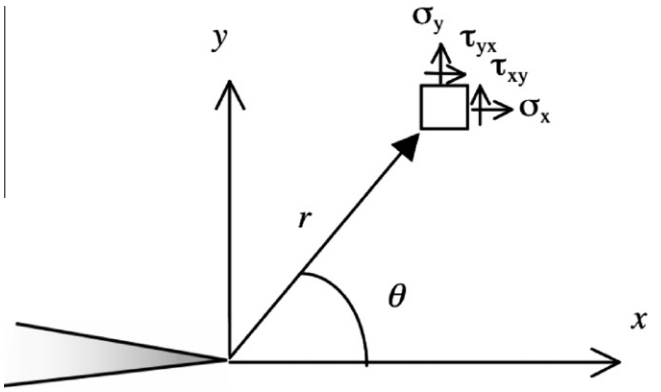

Fig. 1. General mode I problem.

The parameter K_I in these equations is known as the SIF. It is a measure for the stress singularity at the crack tip.

Fig. 2 shows how the concept is used in blistering development in this research. The areas away from the blister are the stress free areas. Near the tip where the blister is propagating is defined as stress singularity area. The pressure difference between the atmospheric pressure and internal pressure of the blister is the driving force for the crack propagation, which in this case is blister propagation.

To enable the use of fracture mechanics concept easily, the weight function method is used here. Only brief discussion is presented here, while the detailed discussion about the concept is available elsewhere [1]. This method converts the internal pressure into a stress intensity factor (see Fig. 3). One of the most important features of the weight function is that it depends only on the geometry of the body and the crack, being independent on the loading. Once the weight function for a given crack body is known, the stress intensity factor for that particular crack tip can be computed directly from any force loading.

The concept of the weight function enables one to calculate the stress intensity factor for a certain loading system by using a reference stress intensity factor for different loading system. It exists for any crack problem specified by the geometry of the component and a crack type. If this function is known, the SIF can be obtained by simply multiplying this function by the stress distribution and integrating it along the crack length.

A crack of length a in a body may be loaded by tractions $\mathbf{T}(s) = (T_y, T_x)^T$ acting on a curve Γ (an internal or external surface), as shown in Fig. 4.

The tractions are responsible for a stress field at the crack tip, which can be characterized by a stress intensity factor K^T, where the superscript "T" refers to the loading system. One can write

$$K^T = \int_\Gamma \mathbf{T}\cdot\mathbf{m}\,ds, \quad (8)$$

where $\mathbf{m}$ is the vector of the weight function, $\mathbf{m} = (m_y, m_x)^T$. Rice has shown that the weight function is related to the displacement field $\mathbf{u} = (u_y, u_x)^T$ under an arbitrary reference load [8] by

$$\mathbf{m} = \frac{H}{K_{ref}}\frac{\partial\mathbf{u}}{\partial a}. \quad (9)$$

With H is the generalized Young's modulus, equals to E for plane stress and $E/(1-\nu^2)$ for plane strain. K_{ref} is the stress intensity factor for the chosen reference loading case. In most practical cases of mode I loading, the stresses along the prospective crack line are of interest. In this case the vectorial quantities reduce to:

$$\mathbf{m} \to m_y(\text{or } h),\ \mathbf{T} \to \sigma_y,\ \text{and } \mathbf{u} \to u_y,\ \text{or written as } v. \quad (10)$$