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Optimal non-periodic inspection for a multivariate degradation model

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Abstract

We address the problem of determining inspection and maintenance strategy for a system whose state is described by a multivariate stochastic process. We relax and extend the usual approaches. The system state is a multivariate stochastic process, decisions are based on a performance measure defined by the values of a functional on the process, and the replacement decision is based on the crossings of a critical levels. The critical levels are defined for the performance measure itself and also as the probability of never returning to a satisfactory level of performance. The inspection times are determined by a deterministic function of the system state. A non-periodic policy is developed by evaluating the expected lifetime costs and the optimal policy by an optimal choice of inspection function. The model thus gives a guaranteed level of reliability throughout the life of the project. In the particular case studied here, the underlying process is a multivariate Wiener process, the performance measure is the ℓ_2 norm, and the last exit time from a critical set rather than the first hitting time determines the policy.

Keywords: Wiener process; Regenerative process; Renewal-reward; Dynamic programming; Statistical testing; Health monitoring

1. Introduction

The model developed is applicable in many contexts but its particular characteristics which guarantee a minimum level of reliability throughout the life of the system are particularly applicable to infrastructure and to safety critical systems.

In most earlier work the system is described by a univariate stochastic process X_t whose performance as represented by X_t must meet some specified requirement. The problem is then usually formulated as repair the system on inspection if its state has not crossed a critical threshold, and to replace if the system's state has exceeded the critical threshold. The policy is defined as a series of inspection instants with a decision rule that determines the action to take after observing the system. Many authors have restricted the modelling to the family of Lévy processes to retain the Markov property. Since a requirement continuity of sample paths restricts the Lévy processes to the non-monotone Wiener process attention has been focussed on retaining monotonicity through the use of jump processes and, in particular, the gamma process [1]. Others have used the Wiener process but generally force almost monotone behaviour by ensuring that the volatility is much smaller than the drift [2]. In both the Wiener and the gamma process the first hitting time distributions for the time to cross a critical threshold are readily obtained. The approach can be extended to non-monotone processes by using the maximum process $M_t = \max_{0 \leqslant s \leqslant t}\{X_s\}$. Because the maximum process is monotone, the apparatus of the standard models becomes available. The use of first hitting times also brings a simplification to the modelling because they are stopping times [3].

In discussions with reliability practitioners it became clear that in many cases a non-monotone process provided a good description of the system's behaviour and one of the concerns was whether a usage threshold had been crossed without causing immediate failure. Furthermore, a system could cross a threshold and then return below either as a result of minor repair or a reduced intensity of use. However, eventually the aging process would drive the system permanently above the critical threshold. We incorporate this possibility in the model developed in this paper.

We further extend and relax the structure of the model to allow for a multi-variate state description, $X_t \in \mathbb{R}^p$. When

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the system is inspected a performance measure is calculated. The performance measure is a functional on the underlying process: $Y_t = A(X_t)$. In this way a bivariate process is used to determine the actions to be taken on inspection [4]. The performance measure is no longer required to be monotone and to simplify the analysis we introduce a different set of criteria for the decision process. The new approach is to define a critical threshold which determines the response to an inspection. Because we now wish to ensure a minimum level of reliability is maintained we set the critical threshold at an acceptable level and examine the probability that the system will never return to this level after crossing it. The idea is that the system may exceed the critical level, but recover back to or below the critical level. Eventually, the system may cross and never return to the acceptable level. When this occurs, the system is aging in such a way that it needs to be repaired or replaced. The decision is thus based on the probability that the system never recovers and if this probability is too small, the system is repaired or replaced. This brings a new aspect to the modelling because this time is not a stopping time. The time of the up-crossing is not a stopping time because we need to now the future of the process if we are to decide that it is the last exit time. In other words it is impossible to know that an observed up-crossing of the threshold level is the last exit time; there remains the possibility of another down-crossing and up-crossing in the future. The difficulty will be seen to be removed by the use of the probability that the last up-crossing occurs between the current time and the next scheduled inspection. The use of the probability on not returning below the current level has a natural interpretation since it corresponds to the probability of a failure in the future and we will control the process to ensure that this probability remains below a specified level. The situation is illustrated in Fig. 1.

The specific model used is based on the assumption that the underlying process is a multivariate Wiener process.

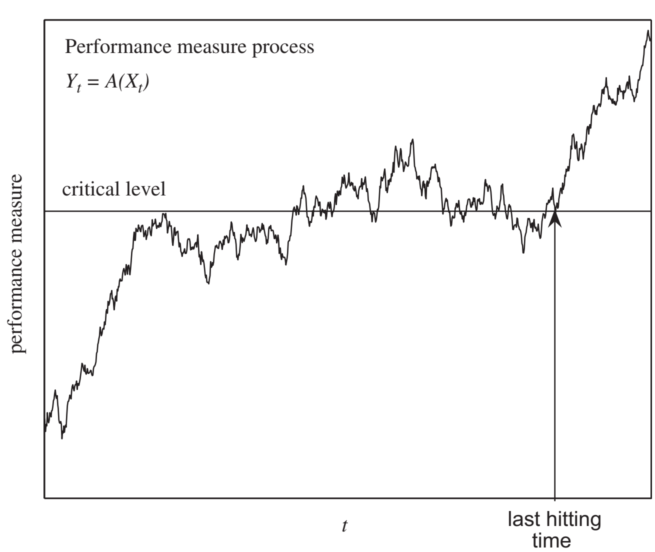

Fig. 1. Exit times.

2. The model

Here we give a the detailed description of the model and the degradation process.

2.1. Notation list

$(B_t)_{t\geqslant 0}$	standard Brownian motion
$(X_t)_{t\geqslant 0}$	stochastic process describing the system's state through time
ξ	critical threshold
H_ξ^0	$\sup_{t\in\mathbb{R}^+}\{t : X_t > \xi \mid X_0 = 0\}$
h_ξ^0	probability density function for H_ξ^0
$V_{\xi-x}^{(0)}$	cost of maintenance per inspection cycle, given that the initial system state is $X_0 = 0$ and that the new critical threshold is equal to $\xi - x$
$v_{\xi-x}^{(0)}$	expected cost of maintenance per inspection cycle, given that the initial system state is $X_0 = 0$ and that the new critical threshold is equal to $\xi - x$
f_t^0	transition density for the Bessel process starting from state $X_0 = 0$, in amount of time t
m	inspection scheduling function

Any other notation used is defined throughout the paper.

2.2. Modelling degradation

The aim of the paper is to derive a cost-optimal inspection and maintenance policy for a complex multi-component system whose state of deterioration is modelled with the use of a Markov stochastic process. Assume that the considered system S consists of N components (or subsystems), each of which experiences its own way of deteriorating through time. Assume further that the N deteriorations are independent: the deterioration of any component has no influence on the deterioration of the $N-1$ remaining components.

The proposed model takes into account the different N deterioration processes as follows. Each component undergoes a deterioration described by a Wiener process. The components are labelled C_i, $i \in \{1,\ldots,N\}$ and the corresponding Wiener processes are $W_t^{(i)}$, $i \in \{1,\ldots,N\}$, where

$$W_t^{(i)} = \mu_i t + \sigma B_t^{(i)} \quad \forall i \in \{1,\ldots,N\},$$
$$W_0^{(i)} = 0 \quad \forall i \in \{1,\ldots,N\}. \tag{1}$$

The above Wiener processes have different drift terms (the μ_i's) but for simplicity the volatility terms (σ) are assumed identical and each component is assumed to be new at time $t = 0$. The independence modelled by considering N independent Brownian motions $B_t^{(i)}$'s.

The next step consists in considering the following N-dimensional Wiener process:

$$\mathbf{W}_t = (W_t^{(1)}, W_t^{(2)}, \ldots, W_t^{(N)}) = \underline{\mu}\, t + \sigma \mathbf{B}_t, \tag{2}$$

$$\mathbf{W}_0 = \underline{0},$$