Theoretical and Applied Mechanics Letters 6 (2016) 81–96

Contents lists available at ScienceDirect

Theoretical and Applied Mechanics Letters

journal homepage: www.elsevier.com/locate/taml

Letter

Elastic properties of chiral, anti-chiral, and hierarchical honeycombs: A simple energy-based approach

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HIGHLIGHTS

- Effects of chirality and hierarchy on elastic response of honeycombs are studied.
- Closed-form relations are derived for elastic moduli and validated using finite element method (FEM).
- Chirality always decreases the stiffness and Poisson's ratio.
- Hierarchical refinement increases the stiffness in hexagon based honeycombs.
- Anti-tetra-chiral structure shows anisotropy, auxeticity, and low shear stiffness.

ARTICLE INFO

Article history:
Received 12 January 2016
Received in revised form
26 February 2016
Accepted 29 February 2016
Available online 11 March 2016
*This article belongs to the Solid Mechanics

Keywords:
Chiral
Hierarchical
Honeycomb
Metamaterial
Auxetic

ABSTRACT

The effects of two geometric refinement strategies widespread in natural structures, chirality and self-similar hierarchy, on the in-plane elastic response of two-dimensional honeycombs were studied systematically. Simple closed-form expressions were derived for the elastic moduli of several chiral, anti-chiral, and hierarchical honeycombs with hexagon and square based networks. Finite element analysis was employed to validate the analytical estimates of the elastic moduli. The results were also compared with the numerical and experimental data available in the literature. We found that introducing a hierarchical refinement increases the Young's modulus of hexagon based honeycombs while decreases their shear modulus. For square based honeycombs, hierarchy increases the shear modulus while decreasing their Young's modulus. Introducing chirality was shown to always decrease the Young's modulus and Poisson's ratio of the structure. However, chirality remains the only route to auxeticity. In particular, we found that anti-tetra-chiral structures were capable of simultaneously exhibiting anisotropy, auxeticity, and remarkably low shear modulus as the magnitude of the chirality of the unit cell increases.

Among the most readily observed topological features in natural structures are chirality [1–5], hierarchy [6–13], and hierarchy of chirality [14,15]. Their abundance in nature can be contrasted with traditional man-made constructions, which often rely on multiple materials selection but relatively simpler micro-geometrical constitution. In recent years, following these topological cues, synthetic metamaterials with non-traditional properties such as negative stiffness [16–18], auxeticity [19–22], and negative thermal expansion [23–25] have been proposed. These characteristics make mechanical metamaterials suitable for applications such as novel prostheses [26], fasteners [27], piezo-composites with optimal performance [28], dome-shaped panels [29,30], and high structural integrity foams [31].

Among this general class of metamaterials, periodic chiral lattices such as the ones shown in Fig. 1 have been shown to possess relatively compliant behavior because of their bending dominated response, while exhibiting considerable multi-axial expansion/contraction under uniaxial loads due to auxeticity [32,33]. These features make them optimal candidates for flexible design applications such as micro-electro-mechanical-systems (MEMS) [19,34,35], aircraft morphing structures [36–43], and as analogues of spokes in non-pneumatic tires [44,45]. In addition, chiral honeycombs have been experimentally and numerically shown to possess Poisson's ratios in the range of $-1 < \nu < 0$. For instance, Alderson et al. [46] studied the in-plane elastic

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http://dx.doi.org/10.1016/j.taml.2016.02.004

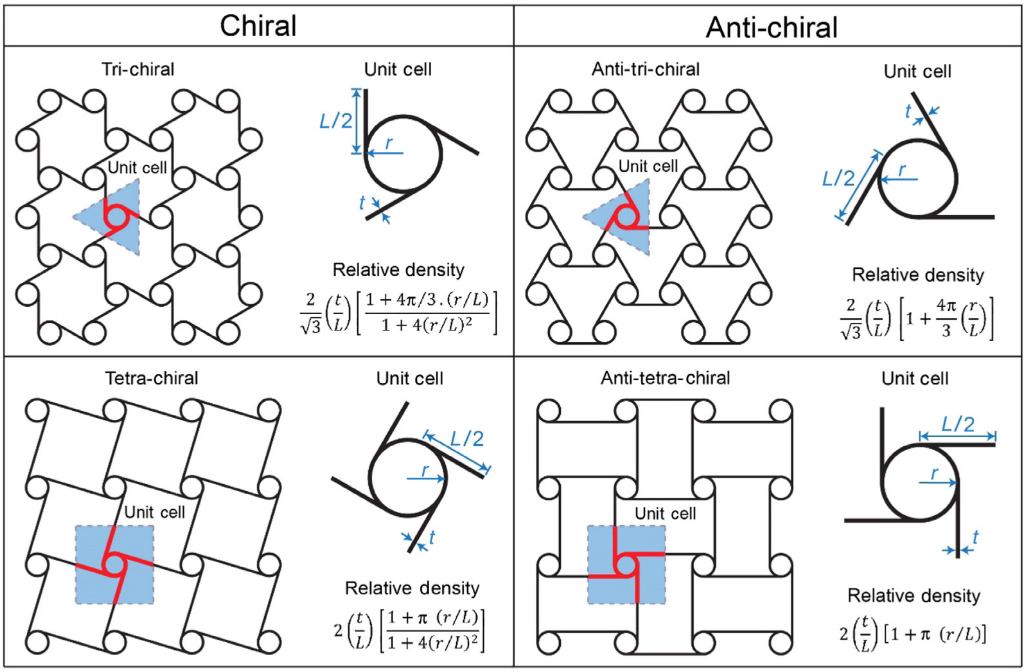

Fig. 1. Schematic of the structure and the unit cell, and the expression of relative density for the chiral and anti-chiral honeycombs studied.

constants of chiral and anti-chiral honeycombs using finite element (FE) analysis and experiments. Continuing further, Alderson et al. [47] investigated the in-plane linear elastic response and out-of-plane bending of tri- and anti-tri-chiral honeycombs and their re-entrant counterparts using FE analysis and experiments. Nonetheless, closed-form expressions of elastic moduli for most of these structures are still unavailable. Among several two-dimensional (2D) chiral lattices proposed in the literature, only the elastic properties of hexa- and tetra-chiral lattices have been investigated analytically, using micro-polar and second-gradient continuum theories [32,48–51]. These approaches are far more complex than the simple yet robust method used here for analytical study of chiral unit cells, which often require special boundary conditions at the unit cell level due to underlying rotational symmetry of the structure.

Another class of bio-inspired materials used increasingly to broaden the achievable range of mechanical response is the hierarchically structured material systems. Extreme values of material properties such as specific stiffness [11,52–54], toughness [55–58], strength [11,53,59,60], buckling strength [61], negative or complex Poisson's ratio [62–65], and phononic band gaps [66] have been reported in hierarchical architectures across multiple length scales. Through a series of studies on the strength of different fractal-like structures under various loads, Farr and co-workers [59,67–70] suggested that the volume of the material used for a stable structure can be reduced by an order of 3–4 under mild loads using hierarchical designs of third and fourth generation. However, the advantage of hierarchical design in these structures diminishes as the magnitude of applied loading increases. Ajdari et al. [52] showed that a type of self-similar hierarchical honeycomb is capable of attaining specific Young's modulus as much as 2 and 3.5 times that of a regular hexagonal lattice through first and second orders of hierarchy, respectively. In a more inclusive study that considered enhancements in multiple parameters, Haghpanah et al. [71] showed that a wide range of specific stiffness and strength can be tailored by introducing higher orders of hierarchy in a hexagonal lattice. However, none of these earlier studies specifically focused on investigating the geometry of hierarchy as a controlling variable of mechanical properties of honeycombs. Moreover, there is no systematic comparison between hierarchy and chirality in the literature, which can be useful in design and selection of structures for different loading conditions.

In light of this discussion, it becomes clear that further investigations on the behavior of these classes of metamaterials are well justified. Particularly, obtaining closed-form analytical expressions for the elastic constants in terms of geometric and material parameters would constitute an important step towards evaluating and designing these materials. Furthermore, it would also foster a better understanding of the role of chirality and hierarchy in influencing the mechanical response of these materials. To this end, in the current paper, we carry out a systematic theoretical and computational study of the effects of these two natural geometrical organizations — chirality and hierarchy — on the in-plane elastic response of 2D honeycombs. In order to directly compare the effects of chirality versus hierarchy, we limit the results to first order of hierarchy for the hierarchical structures presented here. An energy-based method is used to obtain the unit cell deformation by satisfying both the periodic boundary conditions and symmetry requirements for the unit cell. Two specific types of regular tessellation with square and hexagonal cells are altered to endow them with chirality and hierarchy. For achieving chirality, the square based unit cell is altered to yield two different types of chiral architectures — tetra-chiral and anti-tetra-chiral — whereas the hexagonal unit cell alteration results in tri-chiral and anti-tri-chiral structures (illustrated in Fig. 1). In contrast to chiral microstructures, hierarchy is achieved by both conserving the rotational and reflective symmetries of the lattice. This is done by replacing the nodes in a periodic network of cells with the original cells albeit of smaller size as shown in Fig. 2. Thus, the introduction of hierarchy into the square unit cell results in hierarchical square and hierarchical diamond honeycombs (illustrated in Fig. 2). In order to proceed with our calculations, the representative volume element (RVE) is used as the fundamental unit of analysis. In a periodic lattice material, the RVE (i.e., unit cell) is identified as the smallest volume which with associated tractions and displacements, tessellates the space to represent the whole lattice structure under loading [60]. We choose the shaded triangular and square areas bounded by dashed lines in Figs. 1 and 2 as