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Letter

Stability analysis of liquid filled spacecraft system with flexible attachment by using the energy–Casimir method

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HIGHLIGHTS

- The mechanical model of the coupled spacecraft system is constructed.
- The nonlinear stability conditions are obtained by using the energy–Casimir method.
- The stability region of the coupled system is obtained in the parameter space.

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ABSTRACT

The stability of partly liquid filled spacecraft with flexible attachment was investigated in this paper. Liquid sloshing dynamics was simplified as the spring–mass model, and flexible attachment was modeled as the linear shearing beam. The dynamic equations and Hamiltonian of the coupled spacecraft system were given by analyzing the rigid body, liquid fuel, and flexible appendage. Nonlinear stability conditions of the coupled spacecraft system were derived by computing the variation of Casimir function which was added to the Hamiltonian. The stable region of the parameter space was given and validated by numerical computation. Related results suggest that the change of inertia matrix, the length of flexible attachment, spacecraft spinning rate, and filled ratio of liquid fuel tank have strong influence on the stability of the spacecraft system.

The rapid development of aerospace industry requires modern spacecraft to carry large amounts of liquid fuel, and the size of flexible attachments such as the solar panel, antennae, manipulator, is much bigger than before. The motion of the rigid body, liquid fuel, and flexible attachments constituted the complex dynamic system of spacecraft. Take the Cassini–Huygens as an example, which is an unmanned spacecraft sent to the planet Saturn [1]. The spacecraft at launch weighed 5712 kg, which included 3132 kg of propellants. The flexible appendages of the spacecraft contained an 11-meter boom which was used to mount the magnetometer instrument and three other 10-meter rod-like booms which acted as the antennas for the radio plasma wave subsystem. The influence of liquid fuel and flexible attachments should be considered in modeling and analyzing of spacecraft dynamic system, while the weak nonlinear analysis based on perturbation theory was not appropriate for this situation. Related researches [2,3] show that complex nonlinear dynamic behaviors such as static, periodic motion, quasi-periodic motion and chaos will be shown in the coupled spacecraft system, and the types of stable motion in-plane modes and out-plane modes are different when the parameters of the external excitation varied.

Energy–Casimir method can be viewed as a generalization of the classical Lagrange–Dirichlet method and was first proposed by Arnold [4] in studying the stability of stationary flows of perfect liquid. This method was widely used in stability analysis such as the rigid body with flexible appendage [5,6], liquid and plasmas [7]. Casimir function [8] should be used when the energy–Casimir method was adopted in stability analysis. In this way, the conserved quantity can be captured by the Casimir function. In order to overcome the difficulty of constructing the Casimir function, energy–moment method was used to research the stability problems. Energy–moment method is a simplified method because the energy function and the moment map were employed in stability analysis. Simo [9] showed the stability of relative equilibria by the reduced energy–momentum method, and analyzed the nonlinear stability of three dimensional elasticity [10], coupled rigid bodies and geometrically

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exact rods [11]. This method has been extended into stability analysis of the rigid body with a spring–mass particle in the perfect liquid [12], underwater vehicle [13,14], nonholonomic system [15], and plasmas [16].

The stability of the rigid body with flexible attachments can be analyzed by using energy–Casimir method. The dynamic equations of the rigid–flexible coupled system were derived by using Hamiltonian and Poisson bracket, and the stability of the coupled dynamic system was analyzed [6]. The detailed derivations of the nonlinear stability of the rigid body with a linear shearing beam, and the stability conditions of the trivial and untrivial solutions were obtained [5]. The rotation and translation motion of the rigid body with a cantilever beam was discussed by Kane [17], and centrifugal stiffening effect was first proposed. Coupled system which was constituted by a planar rigid body and flexible attachment was studied by Bloch [18], and the nonlinear stability of the equilibria of the equations was discussed. There are also some papers about attitude stability of liquid filled spacecraft. Nonlinear stability of asymmetrical rigid body with full liquid filled satellite was researched by using energy–Casimir method [19], while the self-spinning stability of full liquid filled satellite with flexible appendage was also studied by the same method [20]. The attitude stability of partly liquid filled spacecraft was researched by using energy–Casimir methods, and the liquid sloshing dynamics was simplified as equivalent mass-spring mechanical model [21] and pendulum model [22] to analogue the liquid sloshing dynamics, the stability conditions, and stable region were also given.

In order to meet the precision requirement for modern liquid filled spacecraft with flexible appendages, the impact of rigid–flexible–liquid coupled effect on the spacecraft dynamics and control should be more carefully considered in detail. The equivalent mechanics models and computational fluid dynamics were often used to estimate the dynamic influence of propellant sloshing and attachment vibration on spacecraft [23]. Hybrid-coordinate and spring–mass equivalent model [24], smoothed particle hydrodynamics and absolute nodal coordinate formulation [25] can be utilized to model the spacecraft consisting of a liquid-filled rigid platform and some flexible appendages. There are also some control strategies, such as the variable structure controller [26], robust input shapers [27] for sloshing suppression of the coupled spacecraft system. The attitude maneuver of liquid-filled spacecraft with a cantilever appendage was studied by Yang [28] and the stability criteria of attitude maneuver were derived, while the sloshing liquid was modeled as a viscous pendulum. Related researches mentioned above mainly focused on dynamical modeling and control scheme of the rigid–liquid–flexible coupled spacecraft system. However, it is far away to be completely solved for the rigid–liquid–flexible coupled dynamics problem. For example, as we known, little attention has been devoted to the analytic solution of the stability of partly liquid filled spacecraft system with flexible attachment which plays an important role in the spacecraft dynamics analysis. Energy–Casimir method is an effective method to deal with the stability problem of spacecraft system, and the general stability conditions of the coupled spacecraft system can be obtained.

This paper is concentrated on the stability of the rigid–liquid–flexible coupled spacecraft dynamical system by the energy–Casimir method. The liquid fuel is modeled as a mass–spring mechanical model in order to consider lateral moving in one direction. For linear planar lateral liquid motion, this model is effective to describe linear dynamics of liquid motion and can be used to formulate the dynamic system behavior properly. The flexible attachment is simplified as a linear shearing beam. The framework of this article is this: The mechanical model of the coupled system is given, and dynamic equations and Hamilton function are deduced. Stability conditions were derived by computing the variation of Casimir and energy function. The spin-rotation stability conditions of the coupled spacecraft system were given and the effectiveness of the theoretical derivation was verified by numerical simulation. Related results suggest that the change of inertia moment of rigid spacecraft, the length of beam, and filled ratio of the tank have strong influence on the stability of the coupled system. Conclusions were presented.

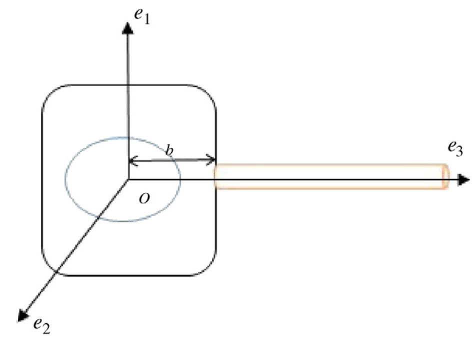

Fig. 1. Dynamic model of liquid filled spacecraft with flexible attachment.

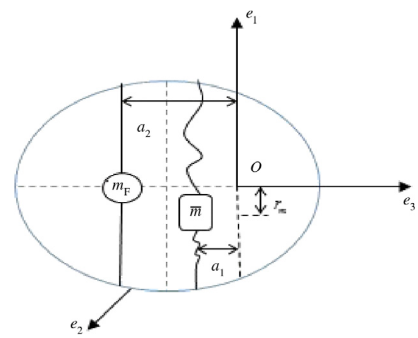

Fig. 2. Equivalent mass–spring mechanical model of liquid fuel in spacecraft.

The mechanical model of the spacecraft system with flexible attachment and ellipsoid tank is illustrated in Fig. 1. In order to research the attitude stability of the coupled system, the body frame is centered at mass center O of the rigid spacecraft. The reference axes $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$ of the body frame are principal axes of rigid spacecraft and $\boldsymbol{J}_\mathrm{H} = \mathrm{diag}(j_{11}, j_{22}, j_{33})$ denotes the inertia matrix of rigid spacecraft respect to the body frame.

The equivalent mechanical model which was modeled as the mass–spring mechanical model is shown in Fig. 2. Mass of sloshing liquid is represented by $\bar{m}$, which is attached to the linear spring. The general position of $\bar{m}$ is $\boldsymbol{r}_{\bar{m}} = (r_\mathrm{m}, 0, a_1)^\mathrm{T}$, while the static position of $\bar{m}$ is $\boldsymbol{r}'_{\bar{m}} = (0, 0, a_1)^\mathrm{T}$. Rest of the fuel mass m_F is regarded as stationary, and its position is denoted by $\boldsymbol{r}_\mathrm{F} = (0, 0, a_2)^\mathrm{T}$.

Assume that $\boldsymbol{S}(\boldsymbol{r}) = \begin{pmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{pmatrix}$ is the skew symmetric matrix of vector $\boldsymbol{r} = (r_1, r_2, r_3)^\mathrm{T}$. We can get $\boldsymbol{r} \times \boldsymbol{b} = \boldsymbol{S}(\boldsymbol{r})\boldsymbol{b}$ for any vector $\boldsymbol{b}$. The mass of rigid spacecraft is expressed by m_H, and the stationary mass of coupled spacecraft is denoted by $m_\mathrm{S} = m_\mathrm{H} + m_\mathrm{F}$. The inertia matrix $\boldsymbol{J}_\mathrm{S}$ of stationary mass respect to the body frame is represented by

$$\boldsymbol{J}_\mathrm{S} = \boldsymbol{J}_\mathrm{H} + m_\mathrm{F}\boldsymbol{S}^\mathrm{T}(\boldsymbol{r}_\mathrm{F})\boldsymbol{S}(\boldsymbol{r}_\mathrm{F}) = \mathrm{diag}(j_{\mathrm{S}1}, j_{\mathrm{S}2}, j_{\mathrm{S}3}), \tag{1}$$

where $j_{\mathrm{S}1} = j_{11} + m_\mathrm{F}a_2^2, j_{\mathrm{S}2} = j_{22} + m_\mathrm{F}a_2^2, j_{\mathrm{S}3} = j_{33}$. The angular velocity of spacecraft is represented by $\boldsymbol{\Omega}$. The velocity of sloshing mass is $\boldsymbol{v}_{\bar{m}} = \boldsymbol{\Omega} \times \boldsymbol{r}_{\bar{m}} + \dot{\boldsymbol{r}}_{\bar{m}}$. Thus, the kinetic energy of the