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Microstructure damage related to stress–strain curve for grain composites

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ABSTRACT

Mathematical model of micro-heterogeneous medium with random deformation and strength properties of microstructure is developed assuming that the tensor of macroscopic deformations is known for the structure. Green–Somigliana tensor is used to obtain the formulas for random stress distribution in microstructure elements. The probability of the stress exceeding the ultimate strength in an element determines the probability of fracture in this element and the relative micro-damage. The correlation functions of stochastic microstructure ultimate strength condition are calculated for various types of stress. Normal distribution is used to calculate the damage. The distribution density can be adjusted through the stress moments to the fourth order.

Micro-fractions change the composite's macro modules of elasticity. Therefore, changes the relationship between stress and strain. Setting an increment step on the macro-strain axis, the stress–strain curve is plotted taking into account changes in composite properties. Stress–strain curves are obtained for different types of load.

The increase of the factor of safety corresponds with the reduction of microstructure damage permitted in the design. Critical microstructure damage also depends on the dispersion of the microstructure properties. It is shown that the microstructure properties of composite significantly influence the behavior of materials under load and the shape of stress–strain curve. Findings are compared with experiment data.

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1. Introduction

Stress–strain curves reflect the relationship between stress and strain in materials under load. With increasing load, the properties of material change. Even under the elastic deformation, there is some damage to the material's microstructure [1–3]. Insignificant at first, micro-fractures develop, build up and shift the material into the zone of plastic deformation. Growing damage changes composite properties and the relationship between stress and strain.

Calculation of macroscopic composite properties of as a function of its microstructure is one of the main tasks of the composite theory [2–4]. Adjustment to the tensor of average elasticity modules is calculated while taking into account the interaction of microstructure elements. Methods for calculating the properties of stochastic composites are improved [5]. This makes it possible to research a range of new types of composites.

In order to solve the stochastic boundary value problem for micro-heterogeneous media it is necessary to find the distribution laws of random strains and stresses in elements of microstructure [2–4]. Such laws are needed to create statistical theories of deformation and to assess the reliability of materials and construction elements. Calculation of strain and stress distribution parameters is associated with significant computational challenges. In practice, the approximate formulas are used, and therefore the accuracy and the applicability range of the methods should be assessed [6].

Microstructure ultimate strength condition ties random stresses and ultimate strengths in the microstructure elements. The probability of stress exceeding the ultimate strength in one element determines the probability of failure of this element and the relative damage at the micro level. Knowing the damage at each stage of the calculations, the changed deformation properties of the material can be determined.

Defining increment step for macro strain axis, draw segments of stress–strain curve taking into account the changed properties on each interval. Mathematical models used to calculate the deformation diagram make it possible to explore a wide range of properties of materials and various types of stresses. The proposed methods also make it possible to solve the opposite problem – based on a given stress–strain curve calculate properties of the composite microstructure with a desired degree of accuracy.

2. Boundary value problem for micro-heterogeneous media

Let us consider a model of micro-heterogeneous medium, containing elements of the first and second orders infinitesimal [3].

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Elements of the first order infinitesimal $\Delta^1 V$ have deterministic macroscopic mechanical properties. The medium is macroscopically homogeneous and isotropic. The size $\Delta^1 V$ is approximately equal δL, where L is a characteristic dimension of the considered part, and $\delta \ll 1$. The size of the microstructure elements $\Delta^2 V$ is by 2–3 orders of magnitude smaller than the size of $\Delta^1 V$. The unit dimension can be chosen as the size of a grain or a fraction of that size. Grain composites have spherical regions of statistical dependence of microstructure properties. The distance at which appropriate correlation functions attenuate defines the radius of the sphere.

Consider the random modulus of volumetric strain, $K(X)$, the shear modulus, $G(X)$, the random Young's modulus, $E(X)$ at a point $X = (x_1, x_2, x_3)$. Corner brackets denote the averaging of random variables. The means of distribution or modules are marked E, K, and G.

$$E = \langle E(X) \rangle; K = \langle K(X) \rangle; \qquad G = \langle G(X) \rangle. \tag{1}$$

Tensor of random elastic modules $\boldsymbol{\Theta}(X)$ can be expressed through the modules $K(X)$ and $G(X)$ or $E(X)$ and the Poisson's ratio ν. Also included are the volumetric component $\mathbf{V}$ and the deviation component $\mathbf{D}$ of the fourth rank unit tensor. Variations of tensor of microstructure elastic modules $\langle \boldsymbol{\Theta}^0(X) \rangle = \boldsymbol{\Theta}(X) - \langle \boldsymbol{\Theta}(X) \rangle$ will depend on variations of the elastic modules $E^0(X)$, $K^0(X)$, and $G^0(X)$. As a result, the two notations of the tensor $\boldsymbol{\Theta}(X)$ are obtained.

$$\boldsymbol{\Theta}(X) = 3K(X)\mathbf{V} + 2G(X)\mathbf{D}, \qquad \boldsymbol{\Theta}^0(X) = 3K^0(X)\mathbf{V} + 2G^0(X)\mathbf{D}, \tag{2}$$

or

$$\begin{aligned} \boldsymbol{\Theta}(X) &= E(X)\left(\frac{1}{1-2\nu}\mathbf{V} + \frac{1}{1+\nu}\mathbf{D}\right), \\ \boldsymbol{\Theta}^0(X) &= E^0(X)\left(\frac{1}{1-2\nu}\mathbf{V} + \frac{1}{1+\nu}\mathbf{D}\right). \end{aligned} \tag{3}$$

It is assumed that for the entire structure and its elements $\Delta^1 V$ the solution of macroscopic boundary problem and the tensor of macroscopic strains $\mathbf{e}$ are known.

For macroscopically isotropic stochastic grain composites, consider the problem of mechanics of micro-heterogeneous media that links the random modules of elasticity tensor $\boldsymbol{\Theta}(X)$, microstructure strains $\boldsymbol{\varepsilon}(X)$ and stresses $\sigma(X)$. Random microstructure stresses and strains are determined through the stochastic boundary problems and the Green's tensor [3,6]. The element $\Delta^1 V$ contains a large enough number of elements of the second order infinitesimal to allow for application of the Green–Somigliana tensor G_{ij} (X, Y). In this case, the integration occurs over an infinite region surrounding the point X.

Operator equation for the main stochastic boundary problem in terms of strains [3,6] is:

$$\varepsilon^0_{ij}(X) = \iiint G_{\varphi(i,j)\psi}(X,Y)\Theta^0_{\varphi\psi\alpha\beta}(Y)\varepsilon_{\alpha\beta}(Y)dV, \tag{4}$$

$i, j, \alpha, \beta, \varphi, \psi = 1, 2, 3.$

A comma before the index denotes the differentiation over the corresponding variable: $f_{,i} = \partial f/\partial y_i$, $f_{,ij} = \partial^2 f/\partial y_i \partial y_j$. From here on the repeated Greek indexes denote summation.

Operator Eq. (4) contains second derivatives of tensor $G(X,Y)$.

$$\begin{aligned} G_{m(i,j)n}(X,Y) &= \frac{1+\nu}{4\pi E}\left(\delta_{mi} r^{-1}{}_{,jn} + \delta_{mj} r^{-1}{}_{,in}\right) - \frac{1+\nu}{8\pi E(1-\nu)} r_{,ijmn}, \\ &\text{for } r^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2. \end{aligned} \tag{5}$$

For convenience, tensor $G_{m(i,j)n}(X,Y)$ can be represented in the matrix form $\widetilde{G}_{pq}(X,Y)$. Indexes will be converted $ij \to p, nm \to q$ according to the rule: $11 \to 1$, $22 \to 2$, $33 \to 3$, $23 \to 4$, $31 \to 5$, $12 \to 6$.

$$\widetilde{G}_{pq}(X,Y) = G_{m(i,j)n}(X,Y) \quad \text{for } p,q = 1,2,\ldots,9. \tag{6}$$

Double scalar product of tensors, the convolution over the two indices denotes «··».

$$(\mathbf{C}\cdot\cdot\boldsymbol{\varepsilon})_{ij} = C_{ij\ \alpha\beta}\varepsilon_{\alpha\beta}, \qquad (\mathbf{G}\cdot\cdot\mathbf{C})_{ij\ km} = G_{ij\ \alpha\beta}C_{\alpha\beta\ km}. \tag{7}$$

In vector notation, the operator Eq. (4) takes the form:

$$\begin{aligned} \varepsilon^0_i(X) &= \iiint \widetilde{G}_{i\alpha}(X,Y)\Theta^0_{\alpha\beta}(Y)\varepsilon_\beta(Y)dV, \\ &\text{for } i, \alpha, \beta = 1, 2, \ldots, 6. \end{aligned} \tag{8}$$

or

$$\boldsymbol{\varepsilon}^0(X) = \iiint \widetilde{G}(X,Y)\cdot\cdot\boldsymbol{\Theta}^0(Y)\cdot\cdot\boldsymbol{\varepsilon}(Y)dV. \tag{9}$$

Eq. (9) is solved with method of successive approximations. The distribution laws for the components of the tensor $\varepsilon^0(X)$ at step $n+1$ are found through the same at the step n for tensor $\boldsymbol{\varepsilon}(X)$. As a first approximation, use microstructure strains $\boldsymbol{\varepsilon}(X)$ averaged over volume $\varepsilon = \mathbf{e}$.

$$\begin{aligned} \boldsymbol{\varepsilon}^0(X)^{(n+1)} &= \sum_{Y \neq X} \widetilde{G}(X,Y)\cdot\cdot\boldsymbol{\Theta}^0(Y)\cdot\cdot\boldsymbol{\varepsilon}(Y)^{(n)}dV, \\ \boldsymbol{\varepsilon}(X) &= \boldsymbol{\varepsilon}^0(X) + \mathbf{e}. \end{aligned} \tag{10}$$

The region of integration is divided into densely packed spherical elements $V_0, V_1, \ldots V_n \ldots$, the diameters of which are equal to an average size of microstructure elements. In the operator (9), replace the integration with the summation over the nodes that are the centers of elementary volumes. The central region V_0 includes the considered point X and provides the singular component of the operator. Summation of the regular components takes place over points Y surrounding the central point X. Depending on the required accuracy, consider several spherical layers around the center point. The calculation over the integration grid nodes is carried out within each cycle of strain approximations. This process produces an error associated with the need to limit the number of approximations and the number of layers surrounding the considered points. Convergence and accuracy of the method of successive approximations depends on the attenuation rate of microstructure properties moment functions [6]. In particular, it is shown that the value of variation of microstructure properties and the Poisson's ratio of the composite influence the convergence of the method.

Fig. 1 depicts the dependence of computation error Δ on the coefficient of variation of Young's modulus k of material for $\nu = 0.3$. With small variations, it is possible to stop at a first approximation, but with the increase of the coefficient of variation k the

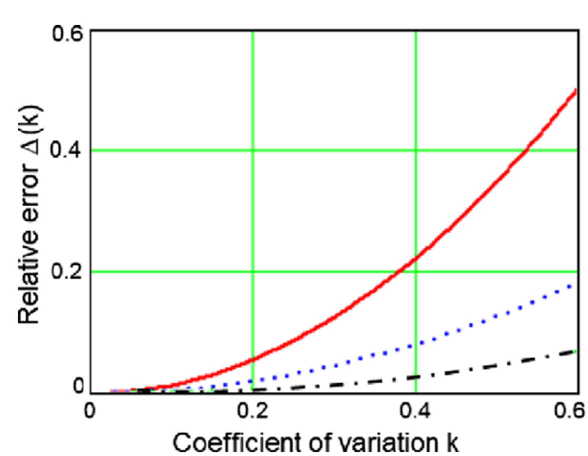

Fig. 1. Relative error $\Delta(k)$ vs. Coefficient of variations k with n approximations for strains.

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