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An extended extreme shock maintenance model for a deteriorating system ☆

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Abstract

In this paper, we study a deteriorating system which is suffering random shocks from its environment. Assume that in the system's operating stage, whenever a shock arrives, it will do some damage to the system, but shocks with a "small" level of damage are harmless for the system, while shocks with a "large" level of damage may result in the system's failure. The system's deterioration is caused by both the external shocks and the internal load. In the external, the magnitude of the shock damage the system can bear is decreasing with respect to the number of repairs taken. In the internal, the consecutive repair time is increasing in the number of repairs taken. A replacement policy N, by which the system is replaced at the time of the Nth failure, is adopted. An explicit expression of the long run average cost per unit time is derived, and an optimal policy N^* for minimizing the long run average cost per unit time is determined analytically. A numerical example is also given.

Keywords: Optimization; Geometric process; Average cost rate; Replacement policy

1. Introduction

In reliability, the study of maintenance problem is always an important topic. In the research work of repair replacement problems, in the early stages, a common assumption is "repair is perfect", i.e., the system after repair is "as good as new". However, it is not always true for a deteriorating system. In practice, most repairable systems are deteriorating because of the ageing effect and accumulated wear.

There are many papers studying maintenance models for the deteriorating systems. Downton [1] and Thompson [2] have used the non-homogeneous Poisson process for modeling deteriorating systems. Barlow and Hunter [3] first introduced a minimal repair model in which the system functions again after repair, but with the same failure rate and the same effective age at the time of failure. Brown and Proschan [4] considered an imperfect repair model, in which a repair is perfect with probability p, and a minimal repair with probability $1-p$. Sheu et al. [5] studied optimum policies for a system with general imperfect maintenance. Kahle [6] studied an optimal maintenance policies in incomplete repair models. In the literature, many papers studied deteriorating systems, Val and Stewart [7], Montoro-Cazorla and Pérez-Ocón [8] and Moustafa et al. [9].

There are also many papers which study optimal maintenance replacement models with shocks. See e.g. Feldman [10], Thompson [2], Stanley [11] and Hu [12].

However, few papers consider the deteriorating systems interrupted by random shocks. In this paper, we discuss a new extreme shock model for a deteriorating system and the system's deterioration is shown in both the internal and the external.

Therefore, we consider the system from two aspects: the internal and the external.

First, if the system is failed by one shock, it is repaired or replaced by a new and identical one. In view of the ageing

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time and the continuous wear, the repair time will become longer and longer and tend to infinity, i.e., finally the system is non-repairable. It is not negligible as some early papers described. See Aven [13], Brown and Proschan [4], and Ascher and Feingold [14]. Therefore, we model the repair times after the system's failures as an increasing geometric process (GP) defined below:

Definition 1. A stochastic process is called a geometric increasing (decreasing) process, if there exists a real $0<a\leqslant 1$ $(b\geqslant 1)$, such that $\{a^{n-1}\xi_n, n=1,2,\ldots\}$ ($\{b^{n-1}\xi_n, n=1,2,\ldots\}$) forms a renewal process (RP). The real $a(b)$ is called the ratio of the GP.

GP was first introduced by Lam [15,16] to study the repair replacement problem for a deteriorating system. Then, using GP, Lam [17] and Stanley [11] studied an optimal repair replacement model for a deteriorating system. The GP has been applied to the study of the optimal replacement problem for a one component and two component series, parallel and standby systems. See Lam [17], Lam and Zhang [18], and Zhang [19]. For more details about GP, refer to Braun et al. [20].

Second, we consider the shocks from the system's environment. There were many papers which considered extreme shock models. See e.g. Zuckerman [21], Shanthikumar and Sumita [22,23], and Gut and Hüsler [24]. In their models, the system will fail if the amount of shock damage by one "big" shock exceeds a specific threshold but a "small" level of damage is harmless for the system. In these models, a shock is called a deadly shock if the amount of damage of one shock to the system exceeds a specific threshold so that the system will fail. This kind of shock models is called "extreme shock model". Finkelstein and Zarudnij [25] studied a shock process with a non-cumulative damage. Lam and Zhang [18] studied a GP maintenance model for a deteriorating system under a random environment, where the shocks reduce the system's operating time but would not cause a failure.

However, for deteriorating systems, the problem is different from those described above. In practice, a deteriorating system after repair should be more fragile and easier to be broken down. As a result, the threshold value which a deadly shock exceeds will be decreasing in n, the number of repairs taken. In other words, the greater the number of repairs adopted, the more fragile the system is. This is a realistic variation of extreme shock model. Consequently, in our model, the threshold value is not a specific value as that of the shock models mentioned above.

Castro and Pérez-Ocón [26] considered a system subject to repairable and non-repairable failures. They studied reward optimization of the system. Moustafa et al. [9] studied optimal major and minimal maintenance policies for semi-Markovian deteriorating systems. In this paper, we study the optimal repairable replacement model for a deteriorating system combined with the new extreme shock model (stated in Section 2). This supplements our results for the cumulative case. See Chen and Li [27]. The aim of the present paper is to provide a method to calculate the average cost rate (ACR) and to determine an optimal policy N^* such that the ACR is minimized analytically and numerically.

2. The model

In repair replacement problems, usually, there are two policies in the context.

Definition 2. A replacement policy T is a policy by which we replace the system whenever the working age of the system reaches T.

Definition 3. A replacement policy N is a policy by which we replace the system at the time of Nth failure since the last replacement.

Now we make the following assumptions about the model for a deteriorating system subject to shocks.

(1) At the beginning, a new system is installed. Whenever the system fails, it is either repaired or replaced by a new and identical one.
(2) Once the system is operating, the shocks from the environment arrive according to a RP. $\{X_{ni}, i=1,2,\ldots\}$ are the intervals between the $(i-1)$st and the ith shock after the $(n-1)$st repair. $\{Y_{ni}, i=1,2,\ldots\}$ is the sequence of the amount of shock damage produced by the ith shock after the $(n-1)$st repair. Let $E[X_{11}]=\lambda$, $E[Y_{11}]=\mu$. We assume that $\{X_{ni}, i=1,2,\ldots\}$ are iid sequences for all n and so are $\{Y_{ni}, i=1,2,\ldots\}$.
    In the nth operating stage, i.e., after the $(n-1)$st repair, the system will fail if the amount of the a shock damage first exceeds $a^{n-1}M$ where $0<a\leqslant 1$. If the system fails, it is closed so that the random shocks have no effect on the system during the repair time.
(3) Let Z_n be the repair time after nth repair. $\{Z_n, n=1,2,\ldots\}$ constitutes a GP with $E[Z_1]=\delta$ and a ratio b, such that $0<b\leqslant 1$. $N_n(t)$ is the counting process of the number of shocks after the $(n-1)$st repair. It is clear that $E[Z_n]=(1/b^{n-1})\cdot\delta$.
(4) Let Z be the replacement time and $E[Z]=\tau$.
(5) The replacement policy N (defined later) is adopted.
(6) The processes $\{X_{ni}, i=1,2,\ldots\}$, $\{Y_{ni}, i=1,2,\ldots\}$, $\{Z_n, n=1,2,\ldots\}$ and Z are independent.
(7) The operating reward rate is r, the repair cost rate is c and the replacement cost is c_r.

Since as stated in Section 1, the threshold value of a deadly shock is decreasing in n, the number of repair adopted or the number of failures occurred. As an approximation, we may assume that the threshold value decreases in n geometrically, i.e., the threshold value of a deadly amount of cumulative shock damage following the nth repair is equal to a^nM with $0<a\leqslant 1$. In the context,