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Effect of transverse cracks on the effective thermal expansion coefficient of aged angle-ply composites laminates

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Abstract

A modified shear-lag analysis, taking into account the concept of stress perturbation function, is developed and applied to evaluate the effect of transverse cracks on the effective thermal expansion coefficient of aged angle-ply composites laminates. Effects of number of 90° layers and number of $\theta°$ layers in the outer angle-ply laminates on the reduction of the effective axial coefficient of thermal expansion have also been studied. The results of this paper represent well the dependence of the reduction of the effective axial coefficient of thermal expansion on the hygrothermal conditions, the fibre orientation angle of the outer layers, the number of cracked cross-ply layers and the number of un-cracked outer $\theta°$ layers in laminate.

Keywords: Transverse cracks; Aged angle-ply laminates; Stress perturbation function; Thermal expansion; Fibre orientation

1. Introduction

In recent years, fibre-reinforced composite materials have been widely used in the aerospace, marine, automobile and other engineering industries. During the operation life, the variation of environmental conditions reduces the elastic moduli and degrades the strength of the laminated composite materials [1–9]. Also the environmental conditions in terms of temperature and moisture induce the hygrothermal stresses within the plies [10–16]. Furthermore, hygrothermal aging in the absence of external loads can lead to spontaneous formation of matrix microcracks in laminates [17]. One of the key features of this material class is their damage initiation and propagation behaviour which, in contrast to monolithic materials, is spatially distributed in nature and comprises a variety of mutually interacting damage modes. The most common damage modes are matrix cracking, delamination growth and fibre fracture.

In particular, matrix transverse cracks appear at a much lower stress than those predicted by classical lamination theory and first ply failure criterion. A review of the available literature results shows that the damage parameter under investigation has always been the longitudinal Young's modulus and the Poisson's ratio [6–9,18–23].

In this work a systematic study will be made of the influence of transverse matrix cracking on the

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evolution of the change of thermal expansion of hygrothermal aged angle-ply composites laminates with crack densities. A general expression for this damage parameter versus transverse crack density is obtained by introducing the stress perturbation function. The effect of fibre orientation angle $\theta°$, in the outer layers on the 90° ply crack behaviour is also studied.

2. Analysis

It is well known in many studies [1–9] that the material properties are function of temperature and moisture. In terms of a micro-mechanical model of laminate, the thermal expansion coefficients in the longitudinal and transverse directions may be written as [24]

$$\alpha_{11} = \frac{V_f E_f \alpha_f + V_m E_m \alpha_m}{V_f E_f + V_m E_m} \tag{1}$$

$$\alpha_{22} = (1 + \nu_f) V_f \alpha_f + (1 + \nu_m) V_m \alpha_m - \nu_{12}\alpha_{11} \tag{2}$$

where α_f and α_m are thermal expansion coefficients of the fibre and matrix respectively.

In the above equations, V_f and V_m are the fibre and matrix volume fractions and are related by

$$V_f + V_m = 1 \tag{3}$$

and E_f, G_f and ν_f are the Young's modulus, shear modulus and Poisson's ratio, respectively, of the fibre, and E_m, G_m and ν_m are corresponding properties for the matrix

$$E_{11} = V_f E_f + V_m E_m \tag{4}$$

$$\frac{1}{E_{22}} = \frac{V_f}{E_f} + \frac{V_m}{E_m} - V_f V_m \frac{\nu_f^2\left(\frac{E_m}{E_f}\right) + \nu_m^2\left(\frac{E_f}{E_m}\right) - 2\nu_f\nu_m}{V_f E_f + V_m E_m} \tag{5}$$

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \tag{6}$$

$$\nu_{12} = V_f \nu_f + V_m \nu_m \tag{7}$$

It is assumed that E_m is a function of temperature and moisture, as is shown in Section 3.1, then α_{11}, α_{22} E_{11}, E_{22} and G_{12} are also functions of temperature and moisture.

2.1. Model formulation

Modified is the progressive shear-lag model [21] by introducing the notion of stress perturbation function [6–9,22]. Consider the idealised cross-ply laminates shown in Fig. 1. When such laminate is

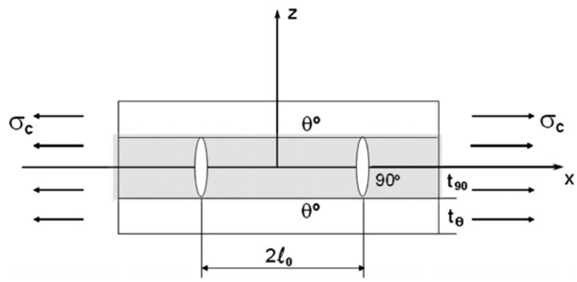

Fig. 1. Transverse cracked cross-ply laminate and geometric model.

loaded in uniaxial tension the first damage which occurs is transverse cracking in the middle layer. The spacing between cracks is assumed to be equidistant, which means that laminate contains a periodical array of cracks in 90°-layer. The geometry of the repeatable unit used for modelling is shown in Fig. 1. Dimensionless coordinates can be introduced:

$$\bar{z} = \frac{z}{t_{90}}; \quad \overline{l_0} = \frac{l_0}{t_{90}}; \quad \alpha = \frac{t_\theta}{t_{90}}; \quad \bar{x} = \frac{x}{t_{90}};$$
$$h = t_\theta + t_{90} \tag{8}$$

Loading is applied only in x-direction and the far field applied stress is defined by $\sigma_c = \frac{1}{2h}N_x$, where N_x is applied load. The following analysis will be performed assuming generalized plane strain condition:

$$\varepsilon_y^\theta = \varepsilon_y^{90} = \varepsilon_y = const \tag{9}$$

The symbol (–) over stress and strain components denotes volume average. They are calculated using the following expressions:

(a) in the $\theta°$ layer.

$$\bar{f}^\theta = \frac{1}{2l_0}\frac{1}{t_\theta}\int_{-l_0}^{+l_0}\int_{t_{90}}^{h} f^\theta \,\mathrm{d}x\,\mathrm{d}z = \frac{1}{2\overline{l_0}}\frac{1}{\alpha}\int_{-\overline{l_0}}^{+\overline{l_0}}\int_{1}^{\bar{h}} f^\theta(\bar{x},\bar{z})\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{z} \tag{10}$$

(b) in 90° layer.

$$\bar{f}^{90} = \frac{1}{2l_0}\frac{1}{t_{90}}\int_{-l_0}^{+l_0}\int_{0}^{t_{90}} f^{90}\,\mathrm{d}x\,\mathrm{d}z = \frac{1}{2\overline{l_0}}\int_{-\overline{l_0}}^{+\overline{l_0}}\int_{0}^{1} f^{90}(\bar{x},\bar{z})\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{z} \tag{11}$$