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Original Article

Finite element formulation and analysis of Timoshenko beam excited by transversely fluctuating supports due to a real seismic wave

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ABSTRACT

Using the concept of quasi-static decomposition and using three-noded isoparametric locking-free element, this article presents a formulation of the finite element method for Timoshenko beam subjected to spatially different time-dependent motions at supports. To verify the validity of the formulation, three fixed-hinged beams excited by the real seismic motions are examined; one is a slender beam, another is a stocky one, and the other is an intermediate one. The numerical results of time histories of motions of the three beams are compared with corresponding analytical solutions. The internal loads such as bending moment and shearing force at a specific time are also compared with analytic solutions. These comparisons show good agreements. The comparisons between static components of the internal loads and the corresponding total internal loads show that the static components predominate in the stocky beam, whereas the dynamic components predominate in the slender one. Thus, the total internal loads of the stocky beam, which is governed by static components, can be predicted simply by static analysis. Careful numerical experiments indicate that the fundamental frequency of a beam can be used as a parameter identifying such a stocky beam.

1. Introduction

The piping in nuclear power plants would experience different motions at its ends during an earthquake if the ends or supports are connected to different main structures. The vibration of the pipe can be treated as the transverse vibration problem of a beam excited by motions at supports when the effect of fluid flow is neglected.

To solve the time-dependent boundary value problems, Mindlin and Goodman [1] developed the solution procedure called “the method of quasi-static decomposition.” They applied it to the problem of the transverse vibrations of Euler–Bernoulli beams with time-dependent boundary conditions. Since then, many investigators have applied it to the analysis of structures subjected to multiple support motions or seismic excitations via various approaches, such as time history analysis, response spectrum method of analysis, or frequency-domain spectral analysis [2–9].

Authors [4,7,10–12] formulated the finite element (FE) dynamic analysis of Euler–Bernoulli beams excited by transversely fluctuating support motions. However, a flexible connecting rod, some robotic manipulators, or some pipes between main structures in nuclear power plants are not slender beams but rather stocky ones. They are often excited by the motions transmitted from connections or supports in main structures or foundation. Thus, it is necessary to develop FE formulation based on Timoshenko beam theory (TBT). However, the investigations for FE formulation of Timoshenko beams subjected to transversely fluctuating support motions are hardly found. As for the analytic solutions of Timoshenko beam excited by transverse support motions, there are not so many articles compared with those concerning analytic solutions of Euler–Bernoulli beam, either. Lee and Lin [13] presented a solution procedure for elastically restrained nonuniform Timoshenko beams by generalizing the quasi-static decomposition method. They [14] also proposed an accurate solution procedure for the forced vibration of a pretwisted Timoshenko beam with time-dependent elastic boundary conditions by using the Mindlin–Goodman's quasi-static decomposition method. Kim [15,16] presented the procedure to obtain the responses of Timoshenko beam excited by support motions by using the expansion theorem based on the orthogonality property of eigenfunctions and also

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presented the analytic solutions of fixed–fixed Timoshenko beam excited by real seismic support motions.

In this study, FE formulation for the dynamic analysis of Timoshenko beam excited by spatially different, transversely fluctuating support motions is presented by using the quasi-static decomposition method. To verify the formulation, FE analysis of the three beams, which are a slender beam, a stocky beam, and an intermediate one, subjected to real seismic time histories of acceleration at supports are performed, and the results are compared with analytic solutions based on the TBT, which are obtained by using the same manner as in the studies by Kim [15] and [16]. The comparisons show good agreements. The static components of bending moment and shearing force of the three beams are also compared with the corresponding total quantities at specific instants at which maximal magnitudes of bending moment and shearing force occur during the excitation. The comparison shows that the static components of the internal loads predominate in the stocky beam, whereas the dynamic components predominate in the slender one. Through careful numerical experiments of nine groups of beams, a parameter identifying which beam is governed by static components is introduced to predict the behavior of such static component–dominated beams (or simply, SCD beam) simply by static analysis, without dynamic analysis.

2. FE formulation

The motion of Timoshenko beam with a uniform cross section in Fig. 1, which is excited by support displacements $a(t)$ and $b(t)$, is described as the following differential equations and boundary conditions.

$$\begin{cases} \rho A\ddot{y} - \kappa GA\left(y_{,xx} - \theta_{,x}\right) = 0 \\ \rho I\ddot{\theta} - EI\theta_{,xx} - \kappa GA\left(y_{,x} - \theta\right) = 0 \end{cases} \tag{1}$$

with the support motions

$$y(x,t)|_{x=0} = a(t), \quad y(x,t)|_{x=L} = b(t), \quad \theta(x,t)|_{x=0} = 0, \quad \theta_{,x}(x,t)|_{x=L} = 0 \tag{2}$$

where $y(x, t)$ is the transverse displacement and $\theta(x, t)$ is the rotational displacement of a cross section. In Eq. (1), a superimposed dot denotes a differentiation with respect to t, and the subscript of x stands for a differentiation with respect to x. EI, ρA, G, and κ denote the flexural rigidity, mass per unit length, shear modulus, and the shear coefficient, respectively. In Eq. (2), $a(t)$ and $b(t)$ denote the prescribed support displacements at the left end and the right end, respectively, and L is the length of the beam.

The free-body sketch in Fig. 2 shows the sign convention of the bending moment and the shearing force. The sign convention will be used in FE formulation later in this article to match up the sign of analytic solutions and the sign of FE solutions. Based on the TBT, the structural loads, $M(x, t)$ and $Q(x, t)$, are expressed as

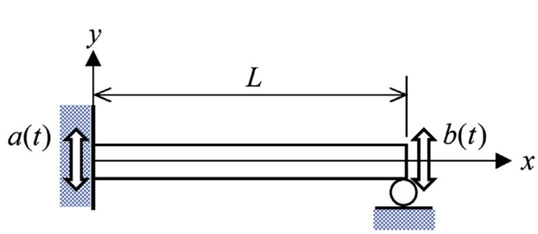

Fig. 1. A fixed-hinged beam subjected to the prescribed displacements $a(t)$ at the left support and $b(t)$ at the right support.

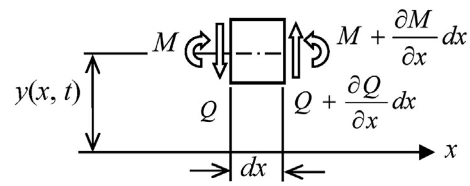

Fig. 2. Free-body sketch of a beam element of length dx.

$$\begin{aligned} M(x,t) &= EI\theta_{,x} \\ Q(x,t) &= \kappa GA\left(y_{,x} - \theta\right) \end{aligned} \tag{3}$$

According to the method of quasi-static decomposition, the total displacements $y(x, t)$ and $\theta(x, t)$ are written as the sum of a static component and a dynamic component, respectively, as follows:

$$\{u\} = \{u_s\} + \{u_d\} \tag{4}$$

where

$$\{u\} = \begin{Bmatrix} y(x,t) \\ \theta(x,t) \end{Bmatrix}, \quad \{u_s\} = \begin{Bmatrix} y_s(x,t) \\ \theta_s(x,t) \end{Bmatrix}, \quad \{u_d\} = \begin{Bmatrix} w(x,t) \\ \phi(x,t) \end{Bmatrix} \tag{5}$$

and y_s and θ_s are static displacements; w and ϕ are dynamic displacements.

2.1. Static displacements

The static displacements, y_s and θ_s, should satisfy the following static governing equations and the support conditions in Eq. (2)

$$\begin{cases} \kappa GA\left(y_{s,x} - \theta_s\right)_{,x} = 0 \\ EI\theta_{s,xx} + \kappa GA\left(y_{s,x} - \theta_s\right) = 0 \end{cases} \tag{6}$$

Solving Eq. (6) directly by using the support conditions, we can obtain the static displacements

$$y_s(x,t) = \frac{b(t) - a(t)}{\gamma + L^2/3}\left(-\frac{x^3}{6L} + \frac{x^2}{2} + \frac{\gamma}{L}x\right) + a(t) \tag{7}$$

$$\theta_s(x,t) = \frac{b(t) - a(t)}{\gamma + L^2/3}\left(-\frac{x^2}{2L} + x\right) \tag{8}$$

where $\gamma = EI/\kappa GA$. The static components of the bending moment and the shearing force are

$$M_{STATIC} = \frac{EI}{\gamma + L^2/3}\left(-\frac{x}{L} + 1\right)\cdot\{b(t) - a(t)\} \tag{9}$$

$$Q_{STATIC} = \frac{EI}{(\gamma + L^2/3)L}\cdot\{b(t) - a(t)\} \tag{10}$$

2.2. Dynamic displacements

Using Eqs. (4) to (6), we can express Eq. (1) in terms of the dynamic displacements, $w(x, t)$ and $\phi(x, t)$, as

$$\begin{cases} \rho A\ddot{w} - \kappa GA\left(w_{,xx} - \phi_{,x}\right) = g_{sy}(x,t) \\ \rho I\ddot{\phi} - EI\phi_{,xx} - \kappa GA\left(w_{,x} - \phi\right) = g_{s\theta}(x,t) \end{cases} \tag{11}$$

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