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Original Article

One-node and two-node hybrid coarse-mesh finite difference algorithm for efficient pin-by-pin core calculation

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ABSTRACT

This article presents a new global–local hybrid coarse-mesh finite difference (HCMFD) method for efficient parallel calculation of pin-by-pin heterogeneous core analysis. In the HCMFD method, the one-node coarse-mesh finite difference (CMFD) scheme is combined with a nodal expansion method (NEM) –based two-node CMFD method in a nonlinear way. In the global-local HCMFD algorithm, the global problem is a coarse-mesh eigenvalue problem, whereas the local problems are fixed source problems with boundary conditions of incoming partial current, and they can be solved in parallel. The global problem is formulated by one-node CMFD, in which two correction factors on an interface are introduced to preserve both the surface-average flux and the net current. Meanwhile, for accurate and efficient pin-wise core analysis, the local problem is solved by the conventional NEM-based two-node CMFD method. We investigated the numerical characteristics of the HCMFD method for a few benchmark problems and compared them with the conventional two-node NEM-based CMFD algorithm. In this study, the HCMFD algorithm was also parallelized with the OpenMP parallel interface, and its numerical performances were evaluated for several benchmarks.

1. Introduction

Accurate analysis of a nuclear reactor core is needed to maximize the core performance and to minimize the uncertainties used in the safety analysis of the reactor system. Currently, the so-called two-step method is exclusively used for the analysis of light water reactor (LWR) cores. In the conventional two-step method, fuel assemblies or subassemblies are analyzed by the accurate transport method to determine homogenized few-group constants; this is followed by a nodal analysis of the three-dimensional whole core using the diffusion theory method with the homogenized group constants [1,2]. In spite of the successful application of two-step procedures in LWR core analysis, it is still necessary to improve the accuracy of conventional two-step methods to minimize the core design uncertainties of both modern LWRs and the more challenging advanced LWR designs.

Recently, to improve the accuracy of reactor core design and analysis, significant efforts have been made for the development of direct whole-core transport calculations without the preanalysis of the lattice [3–7]. Although this whole-core transport approach can provide accurate results, it is very costly in terms of computing time and memory requirements. Another possible way to improve the accuracy of the reactor analysis is to do a pin-by-pin core analysis, in which only the small fuel pins are homogenized, and the three-dimensional core analysis is still performed by low-order methods such as diffusion theory [1,2,8–10]. Therefore, the pin-by-pin core analysis can directly provide detailed core power distributions without any need to use the approximate pin power reconstruction procedure, which is commonly required in the conventional two-step method.

Pin-wise core calculation is a lot more tractable than whole-core transport calculations and is generally known to provide higher accuracy than the standard two-step approaches. However, the pin-wise approach is still computationally intensive because there are a lot more unknowns to be considered, e.g., 17×17 meshes per fuel assembly versus one or four meshes per fuel assembly in the nodal methods. For an efficient pin-wise core calculation, the one-

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node coarse-mesh finite difference (CMFD) method [9] has been developed, in which the global CMFD equations are formulated based on local fine-mesh finite-difference method (FDM) solutions for pin-wise heterogeneous fuel assemblies. The one-node CMFD method is a very promising method because local problems can be solved independently or in parallel. However, its parallel performance has not been investigated by the original developers.

In the recent works by the authors [11,12], the computing efficiency of the one-node CMFD method was substantially enhanced by solving the local problems with the well-developed nodal expansion method (NEM)–based two-node CMFD method [13,14]. Consequently, the one-node CMFD method can now be coupled with the conventional two-node nodal CMFD method. This two-level combination of the two CMFD methods has an unsurpassable advantage in that each local problem can be solved in parallel and efficiently; more importantly, the parallelism is extremely high because most of the computing work is for local problem analysis.

In this article, we present the one- and two-node hybrid CMFD method, called HCMFD, in detail for pin-by-pin whole-core calculation, and its numerical characteristics and performance are evaluated in both serial and parallel platforms for several benchmark problems. In addition, a comparison with the standard two-node CMFD method is made. We evaluated the parallel performance of the HCMFD algorithm on an OpenMP [15] parallel platform.

2. One-node CMFD for global domain

2.1. Basic theory

The one-node CMFD module solves the global neutron balance equation for the whole reactor domain. The global CMFD module provides the eigenvalue and average partial currents at large coarse-mesh boundaries, which are used in the local two-node NEM CMFD and the two-level hybrid algorithm. The main framework of one-node CMFD is expressed in Eq. (1).

$$A_{CMFD}^{\text{global}}\phi_{CMFD}^{\text{global}} = \frac{1}{k}F_{CMFD}^{\text{global}} \Leftrightarrow \left\{ \mathrm{M}_{\mathrm{i}}^{\text{local}}\phi_{\mathrm{i}}^{\text{local}} = \mathrm{S}_{\mathrm{i}}^{\text{local}}, \mathrm{i} = 1,2,\ldots \mathrm{N} \right\} \quad (1)$$

In the nodal equivalence theory, nodal method based on coarse mesh can provide the reference nodal quantities of the neutron diffusion equation when the surface net currents are preserved. In the one-node CMFD method, the fine-mesh heterogeneous local problem, which is a high-order method, is accelerated by a balancing equation based on the coarse-mesh global problem in a nonlinear iteration scheme. The low-order equation comprises homogenized parameters based on equivalence theory. Therefore, the preservation of the reference net currents at the interface can make the one-node CMFD method reproduce reference nodal quantities.

The right side of Eq. (1) shows the local calculation of the one-node CMFD method with fine mesh. The local problem, which is a fixed source problem, is solved by a high-order method, i.e., the nodal method or the transport method. The solution of the local domain produces correction factors which correct the surface current values. In the one-node CMFD method, the local domain is calculated independently with the incoming partial current boundary conditions, which are calculated from the global domain. Therefore, the one-node CMFD method can provide the best computational framework for a parallel global-local nonlinear iteration.

The one-node CMFD equations can be derived from the two-dimensional neutron diffusion equation, which is integrated over the spatial volume of a mesh cell as follows:

$$J_g^{\text{Top}} + J_g^{\text{Down}} + J_g^{\text{Right}} + J_g^{\text{Left}} + \Sigma_{r,g}^{i,j}\overline{\phi}_{g,i,j}\Delta x_i^{\text{global}}\Delta y_j^{\text{global}} = \left(\sum_{g=1}^{G} \frac{\chi_g}{k_{eff}} \nu_g \Sigma_{f,g}\overline{\phi}_{i,j,g} \right) \Delta x_i^{\text{global}}\Delta y_j^{\text{global}}. \quad (2)$$

The direction of each node's net current is outward from the center of the mesh.

As shown in Fig. 1, at each interface of the coarse-mesh unit cell (i,j), the two FDM approximations for the net current are expressed in Eqs. (3a) and (3b).

$$J^{\text{global},+\varepsilon} = -\frac{2D_{i+1,j}}{\Delta x_{i+1}^{\text{global}}}\left(\overline{\phi}_s - \overline{\phi}_{i+1,j}\right), \quad (3a)$$

$$J^{\text{global},-\varepsilon} = -\frac{2D_{i,j}}{\Delta x_{i}^{\text{global}}}\left(\overline{\phi}_s - \overline{\phi}_{i,j}\right), \quad (3b)$$

where

D:diffusion coefficient,
ε:infinitesimally small value,
$\overline{\phi}$:volume – averaged flux,
$\overline{\phi}_s$:suface flux at the boundary

A fine mesh is clearly needed to solve Eq. (2) with good accuracy. However, this would unfortunately require larger computing time than when the problem is solved using a coarse mesh. To reduce computing time, the CMFD method is widely used as an efficient acceleration scheme of the nodal method. It should be noted that, unlike the conventional CMFD, the one-node CMFD formulation uses two correction factors instead of one. With two correction factors, the interface net currents can still be corrected to a reasonable accuracy even when the problem is solved using a coarse mesh. The two net currents in Eqs. (3a) and (3b) can be corrected with two correction factors in the following way:

$$J^{\text{global},+\varepsilon} = -\frac{2D_{i+1,j}}{\Delta x_{i+1}^{\text{global}}}\left(\overline{\phi}_s - \overline{\phi}_{i+1,j}\right) - \frac{2\widehat{D}^{+\varepsilon}}{\Delta x_{i+1}^{\text{global}}}\left(\overline{\phi}_s + \overline{\phi}_{i+1,j}\right), \quad (4a)$$

$$J^{\text{global},-\varepsilon} = -\frac{2D_{i,j}}{\Delta x_{i}^{\text{global}}}\left(\overline{\phi}_s - \overline{\phi}_{i,j}\right) - \frac{2\widehat{D}^{-\varepsilon}}{\Delta x_{i}^{\text{global}}}\left(\overline{\phi}_s + \overline{\phi}_{i,j}\right), \quad (4b)$$

where $\widehat{D}^{\pm\varepsilon}$ are two different correction factors.

Based on diffusion theory, these two currents at the interface are the same. Hence, the two-sided current limits in Eqs. (4a) and (4b) can be made equal to solve the surface flux at the interface. This

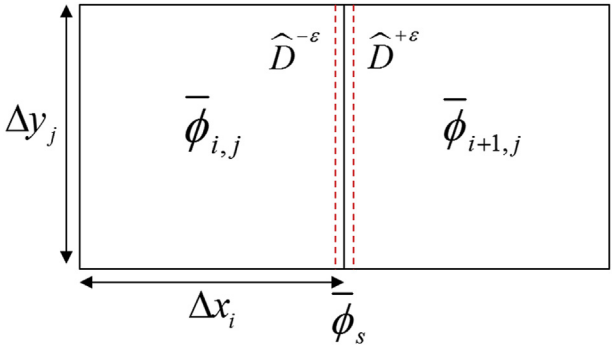

Fig. 1. Two meshes in a global problem.