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A composite spatial grid spectral Green's function method for one speed discrete ordinates eigenvalue problems in two-dimensional Cartesian geometry

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ABSTRACT

Spectral Green's function nodal methods (SGF) are well established as a class of coarse mesh methods. For this reason, they are widely used in the solution of neutron transport problems in discrete ordinates formulation (S_N). When compared with fine mesh methods, SGF are considered efficient, as solutions are as accurate as, using a smaller number of spatial nodes, reducing floating point operations. However, the development of spectral-nodal methods for $X-Y$ Cartesian geometries, has been limited due to (a) difficulties in implementing efficient computational algorithms and, (b) high algebraic and computational costs. This is because these methods need to use NBI-type (One-Node Block Inversion) sweep schemes. The composite spatial grid methods were developed to overcome these challenges. In this work, we describe a composite spatial grid spectral-nodal method to solve one-speed discrete ordinate eigenvalue problems in X−Y Cartesian geometry with isotropic scattering. The discretization is developed into two stages and two 1D problems coupled by transverse leakage terms in each domain region are obtained. In order to converge toward the numerical solution, we used an alternating-direction iterative technique and a modified source iteration sweep scheme. Also, we used the conventional power method to estimate the problem's dominant eigenvalue. Numerical results for benchmark problems are presented to illustrate the accuracy and performance of the developed method. This approach offers more accurate and efficient results for integral quantities if compared with others SGF methods.

1. Introduction

Eigenvalue problems appear in nuclear reactor criticality calculations, where reactor core multiplicative parameters for different geometric configurations, multiples nuclear fuels and several operation conditions are evaluated. On the nuclear reactor core, neutron generation and losses must be in balance during operation. Neutron losses include the leakage outside the core and the absorption by non-fissile materials. On the other hand, neutron generation is due to fission chain reactions. This balance is expressed through the neutron transport equation (Bell and Glasstone, 1970).

The mono-energetic discrete ordinates formulation (S_N) (Lewis and Miller, 1984) is one of the most used models in deterministic calculations of neutron transport. Several methods are used in the discretization of the spatial variables. They are classified as fine (Lewis and Miller, 1984), medium (Wareing et al., 2001; Courtot, 1981) and coarse (Walters, 1986; Lawrence, 1986; Garcia and Siewert, 1981) mesh methods. Coarse mesh methods for spatial discretization are desirable due to their computational efficiency and numerical accuracy. Among them, nodal methods are used to numerically solve the S_N formulation in Cartesian geometry (Badruzzaman, 1990). Nodal methods are supported by the transverse integration procedure and consider approximations for scattering source and leakage terms. Thus, it is possible to use polynomial approximations in all terms, such approach is used by polynomial-nodal methods (Azmy, 1988).

On the other hand, it's possible to treat the source terms analytically and to approximate the transverse leakage terms with polynomials, this strategy is used by the spectral Green's function methods (Barros and

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Larsen, 1992; Barros et al., 1999). The spectral Green's function methods lead to more accurate numerical solutions than polynomial-nodal methods as the scattering source terms are not approximated.

As the total computing time for iterative numerical schemes is proportional to the mesh size, SGF methods are very important class of methods. They generate accurate results for coarser meshes, offering good computational performance when compared with non-SGF methods. Yet, the developed spectral Green's function methods, particularly the Spectral Diamond-Spectral Green's Function-Constant Nodal (SD-SGF-CN) method (Filho et al., 2002) for eigenvalue problems in $X-Y$ geometry, presents difficulties in the computational algorithm implementation and also, high algebraic and computational costs. This is because they use the One-Node Block Inversion (NBI) (Barros and Larsen, 1992) sweep schemes, leading to complex computational algorithms.

In order to avoid the Spectral Green's Function methods' disadvantages, the Composite Spatial Grid-Spectral Green's Function-Constant Nodal (CSG-SGF-CN) method was proposed for fixed source problems (Dominguez et al., 2010). Composite Spatial Grid's methods are characterized for the accurate integral quantities results.

These methods discretize spatial variables by reducing the 2D problem in two 1D problems coupled by the transverse leakage terms through two steps. Firstly, a rectangular coarse mesh with each cell matching a material region is used. Then, for each mesh cell transversely integrated S_N equations in the x and y spatial directions are obtained. In these equations, constant approximations for the transverse leakage terms are introduced. Secondly, a finer spatial grid is used to discretize the transversely integrated S_N equations that can be solved separately. As 1D problems are coupled by the transverse leakage terms, an alternating-direction iterative technique is applied to reach convergence. These methods preserve the spectral-nodal technique's accuracy using simpler one-dimensional formulations.

Following CSG-SGF methods, the Composite Grid Diffusion-Spectral-Green's Function method was developed for eigenvalue problems in $X-Y$ geometry with constant approximation (Nascimento and Dominguez, 2015). Thereafter, the Composite Spatial Grid Spectral Green's Function Exponential method (Aguiar et al., 2015) was introduced which uses exponential approximations and solves fixed source problems.

In this work, we propose a new approach for the family of Composite Grids Spectral-Nodal methods. The development of the Composite Spatial Grid-Spectral Diamond-Constant Nodal (CSG-SD-CN) method solves S_N mono-energetic eigenvalue problems in $X-Y$ geometry with isotropic scattering. The new method extends the Spectral Diamond-Spectral Green's Function method (de Abreu et al., 1996) and uses a constant approximation for the transverse leakage terms. In order to converge to the numerical solutions of the angular flux, we used a modified Source Iteration (SI) sweep scheme (de Abreu et al., 1996), that led to a simpler algorithm than the NBI scheme. We also use the Power method (Burden and Faires, 2010) to estimate the problem's dominant eigenvalue.

In the next section, fundamentals of the CSG-SD-CN method are summarised. In Section 3, numerical benchmarks are solved and numerical results comparing the proposed method with SD-SGF-CN method (Filho et al., 2002) are presented. In Section 4, conclusions are drawn and suggestions for future work are outlined.

2. The CSG-SD-CN method

The present section is divided into three parts. In Section 2.1, the procedure to discretize the spatial variables of the S_N formulation using an external spatial coarse grid is described, *i.e.,* the first step of spatial discretization. In Section 2.2, we briefly present the spectral analysis of the S_N equations transversely integrated to obtain an expression for the general analytic solution in the spatial cell interior with a constant approximation for the transverse leakage terms. In the third and last subsection, we describe the second step of the spatial discretization, this is the procedure to discretize the one-dimensional S_N transversely integrated equations using an internal mesh finer than the external.

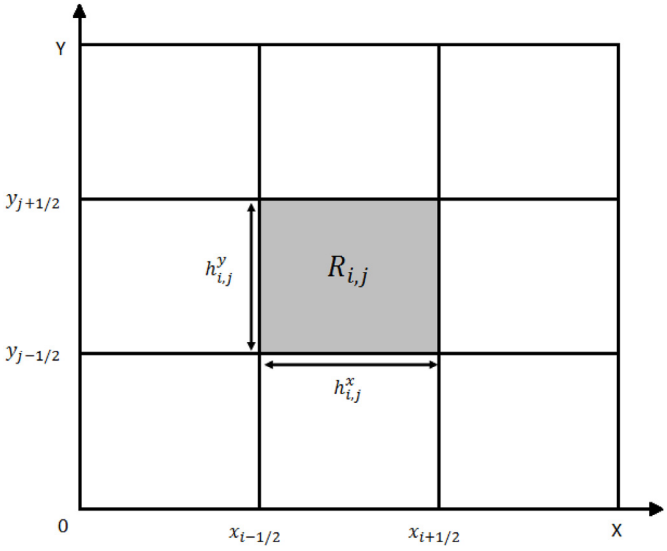

Fig. 1. CSG-SD-CN method external mesh over the problem's 2D rectangular domain.

2.1. External discretization

Consider a 2D rectangular domain in $\mathbb{R}^2$ with dimensions (X, Y), and several rectangular regions $R_{i,j}$, $i = 1{:}\, I$ and $j = 1{:}\, J$. The $R_{i,j}$ regions have constant physical properties and dimensions $(h^x_{i,j}, h^y_{i,j})$, as shown in Fig. 1.

In the first step of the spatial discretization, we introduce a uniform spatial mesh Λ, in which $\Lambda_{i,j}$ cells match with material regions $R_{i,j}$ in Fig. 1, *i.e.*, $\Lambda_{i,j} \equiv R_{i,j}$, for $i = 1{:}\, I$ e $j = 1{:}\, J$. The S_N discrete ordinates formulation (Lewis and Miller, 1984) in each cell appears as,

$$\mu_m \frac{\partial}{\partial x}\psi_m(x,y) + \eta_m \frac{\partial}{\partial y}\psi_m(x,y) + \sigma_{t_{i,j}}\psi_m(x,y) = \frac{\sigma_{s0_{i,j}}}{4}\sum_{n=1}^{M}\psi_n(x,y)\omega_n + \frac{\sigma_{f_{i,j}}}{4k_{eff}}\sum_{n=1}^{M}\psi_n(x,y)\omega_n, \tag{1}$$

where $(\mu_m, \eta_n,$ and $\omega_m)$ are the level symmetry quadrature parameters, ψ_m is the angular neutron flux, $\sigma_{t_{i,j}}$ is the total cross-section, $\sigma_{s0_{i,j}}$ represents the differential scattering cross-section, $\sigma_{f_{i,j}}$ is the fission cross-section and k_{eff} is the effective multiplication factor. Also, $(x, y) \in R_{i,j}$, $m = 1{:}\, M$, $M = N(N+2)/2$ and N is the quadrature order.

In order to reduce the 2D partial differential equations problem (represented by Eq. (1) into two 1D ordinary differential equations problem), we apply the transverse integration concept in x and y directions in each spatial cell $\Lambda_{i,j}$ of the external mesh.

The 1D equations are obtained by applying the following transverse integration operators in one arbitrary cell $\Lambda_{i,j}$ for the y and x spatial directions, respectively,

$$\frac{1}{h^y_{i,j}}\int_{y_{j-1/2}}^{y_{j+1/2}}\cdot dy, \quad \frac{1}{h^x_{i,j}}\int_{x_{i-1/2}}^{x_{i+1/2}}\cdot dx. \tag{2}$$

As the mathematical procedure is similar for each spatial direction, from now on, we assume just the transverse integration equation in y direction and therefore, dependent on x, to illustrate the CSG-SD-CN method construction. Thus, the transverse integrated equation in y direction for the angular flux zero-order moment in one arbitrary angular direction is,

$$\mu_m \frac{d}{dx}\widetilde{\psi}_{m,j}(x) + \sigma_{t_{i,j}}\widetilde{\psi}_{m,j}(x) = \frac{\sigma_{s0_{i,j}}}{4}\sum_{n=1}^{M}\widetilde{\psi}_{n,j}(x)\omega_n + \frac{\nu\sigma_{f_{i,j}}}{4k_{eff}}\sum_{n=1}^{M}\widetilde{\psi}_{n,j}(x)\omega_n - \frac{\eta_m}{h^y_{i,j}}[\psi_m(x, y_{j+1/2}) - \psi_m(x, y_{j-1/2})], \tag{3}$$

where we defined