Progress in Nuclear Energy 108 (2018) 281–285

Contents lists available at ScienceDirect

Progress in Nuclear Energy

journal homepage: www.elsevier.com/locate/pnucene

The cost-benefit analysis of the optimal Type-I (α) and Type-II (β) error values for nuclear material accounting and safeguards

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A R T I C L E I N F O

Keywords:
Nuclear safeguards
Nuclear material accountancy
Cost-benefit analysis
Bayesian decision theory
Type-I error
Type-II error

A B S T R A C T

New methodologies for rationally safeguarding a nuclear system in terms of economics have been developed based on the Bayesian decision theory and the risk management approach. Two methodologies are formulated and presented. In the first method, the two terms in which the cost for an error (Type-I or Type-II error) event is multiplied by the probability of that event occurring and are linearly combined. Second method is formed by utilizing the ratio of two types of risks for each type of error event. The two methods proposed generate adjusted and optimized S (detection threshold) values under given conditions and depending on the expectation of costs for error events. The results show that the methods suggest adjusted S values in terms of the risk and the cost-benefit based on the error values. For instance, the Type-II error event is more forbidden for the system that treats more special nuclear materials (e.g. Pu) during a short period. Moreover, the long-time period without inspection could enhance opportunities for the Type-II error event occurring. These methods could contribute to a better understanding for managing the Type-I and Type-II error events in a nuclear system which should be safeguarded.

1. Introduction

Nuclear material safeguards is a powerful method for nuclear non-proliferation (International Atomic Energy Agency, 2001). For nuclear safeguards, material accountancy is a key factor (International Atomic Energy Agency, 2001). Material accounting activities can be conducted either by facility operators or by the State System of Accounting for and Control of nuclear material (SSAC) (International Atomic Energy Agency, 2001, 2008). The International Atomic Energy Agency (IAEA) report introduces various Non-Destructive Assay (NDA) and Destructive Assay (DA) methods for material accounting and inspection (International Atomic Energy Agency, 2003).

Safeguards for used nuclear fuel and nuclear power plants have been discussed in the past (Durst et al., 2007; Woo and Lee, 2011). One of the approaches for used fuel reprocessing, such as pyroprocessing, is a High Reliability Safeguards (HRS) approach (Borrelli, 2014a, 2014b). The HRS employs a risk-informed approach to minimize the probability of Type-I error (a false positive error). The statistical concepts and descriptions for the Type-I error have been shown in IAEA reports (International Atomic Energy Agency, 1982, 2001) and in the books for the material accountability and the non-proliferation (Avenhaus, 1977; Morse, 2016).

According to Chang's paper (Chang et al., 2011), a desirable Type-I error (false alarm) is 5% for the plutonium (Pu) measurements. However, there is no discussion in the determination on how to achieve a Type-I error of 5%. Therefore, in this study, a methodology to appropriately recommend the probability of Type-I and Type-II error values for a nuclear facility is discussed. The quality control emanating from the Bayesian decision analysis and the cost benefit analysis approaches are referred in this study (Albert, 1978; James et al., 1996; Grosser and Goodman, 1985; Faber, 2003; Leon-garcia, 2008). Those methods have been wieldy applied not only in the nuclear material accounting and safeguards field but also in other technical fields. The objective of this study is to develop appropriate methodologies to investigate an optimum Type-I and Type–II error values for nuclear material accounting and nuclear material safeguards by utilizing the cost-benefit analysis and the risk assessment approaches.

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https://doi.org/10.1016/j.pnucene.2018.06.005
Received 18 January 2018; Received in revised form 9 May 2018; Accepted 6 June 2018

2. Theoretical background

2.1. MUF

For nuclear material accounting in a Material Balance Area (MBA), which is the nuclear material accounting area for reporting to the IAEA (International Atomic Energy Agency) (International Atomic Energy Agency, 2008), operators or inspectors continuously or periodically measure nuclear materials using various instruments. However, these instruments have several types of inherent or external sources of error. The amount of missing material in a MBA is assessed as Materials Unaccounted For (MUF) given by

$$\mathrm{MUF} = (\mathrm{PB}+\mathrm{X}-\mathrm{Y}) - \mathrm{PE}. \tag{1}$$

where PB is the beginning physical inventory of the nuclear material, X is the sum of increases to inventory, Y is the sum of decreases from inventory, and PE is the ending physical inventory.

2.2. Hypothesis testing

The null (H_0) and alternative hypothesis (H_a) are defined as:

$$H_0{:}E[\mathrm{MUF}] = 0, \tag{2}$$

$$H_a{:}E[\mathrm{MUF}] = M, (\ M>0). \tag{3}$$

The non-zero expected value of MUF, M, for the alternative hypothesis, H_a, should be less than one Significant Quantity (SQ; i.e. 1SQ for Pu is 8 kg) (International Atomic Energy Agency, 2001). The probability of Type-I (α) and Type-II (β) errors are given by

$$\alpha = P\{\mathrm{MUF}>S|H_0\} = 1-\Phi\left(\frac{S}{\sigma_{MUF}}\right), \tag{4}$$

$$\beta = P\{\mathrm{MUF}\le S|H_a\} = 1-\Phi\left(\frac{M-S}{\sigma_{MUF}}\right). \tag{5}$$

where S is the detection threshold value and Φ is the cumulative Gaussian distribution function with M as the mean and σ_{MUF} as the standard deviation in MUF measurement. In order to satisfy the Type-I and Type-II errors less than and or equal to 5%, the variable in the brackets of the cumulative Gaussian distribution function in Equations (4) and (5) should be both 1.65. By solving simultaneous equations for M equal to 8 kg, the$\sigma_{MUF}$is 2.42 kg. The variations of Type-I and Type-II errors for a parametric variation of S value are shown in Fig. 1. These parametric variations are provided for two values of σ_{MUF} 2 kg and 2.42 kg to show the relationship between $\sigma_{MUF}$and S. For this parametric study the assumed value of M is 8 kg, which is the value of one SQ for Pu. S and σ_{MUF} contribute together to evaluate the accuracy of material accountancy in a system. From Fig. 1, it can be inferred that for Type-I and Type-II errors to be less than 5% S should be equal to 4 kg when $\sigma_{MUF} = 2.42$ kg. However, when $\sigma_{MUF} = 2.0$ kg, there is a range of S values (3.29 kg–4.71 kg) that could satisfy Type-I and Type-II errors to be less than 5%. It is worth pointing out here that there is no discussion or reason why 5% Type-I and Type-II error probability values are acceptable in nuclear material accountancy applied to safeguards.

3. Methodologies for decision making

This section shows two methodologies to evaluate appropriate probabilities of Type-I and Type-II error. The methods start from adjusting S values when σ_{MUF} is constant at 2.42 kg, even though σ_{MUF} can vary based on a facility and a measurement method.

3.1. Cost-benefit analysis using a linear combination of two error events (Method-1)

Four outcomes can be expected by the hypothesis testing such as a true positive event, a false positive event (α), a true negative event, and a false negative event (β). Costs for those four outcomes can be assigned to each event as a measure of its relative importance (Leon-garcia, 2008). The cost terms for each of the aforementioned four events are defined as:

- C_{00}: the cost of choosing H_0, given that the true is H_0,
- C_{0a}: the cost of choosing H_0, given that the true is H_a,
- C_{a0}: the cost of choosing H_a, given that the true is H_o,
- C_{aa}: the cost of choosing H_a, given that the true is H_a.

Then, the expected cost for all possible events is a linear combination given by

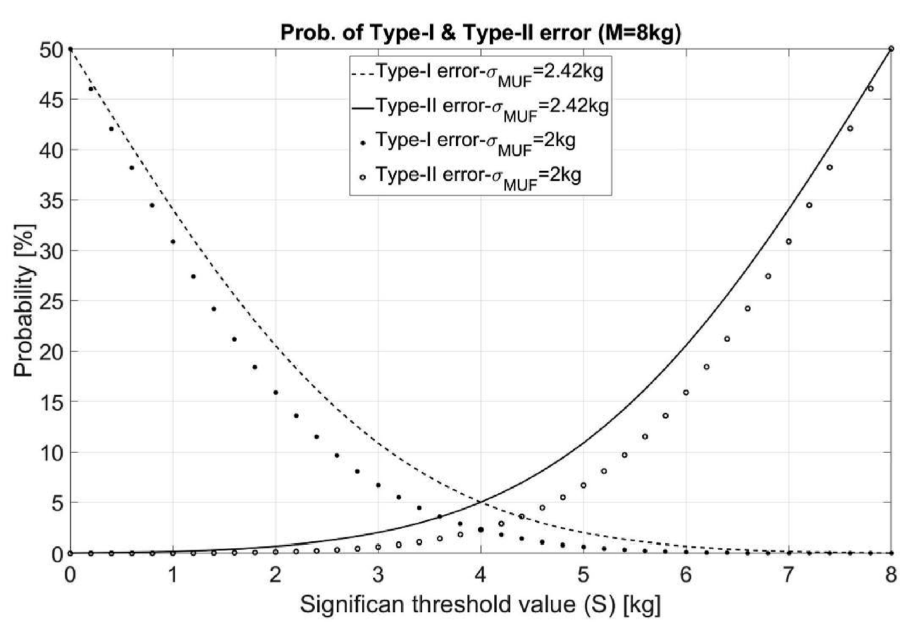

Fig. 1. Probabilities of Type-I and Type-II error as a function of the S value.