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Whole-core forward-adjoint neutron transport solutions with coupled 2-D MOC and 1-D SN and kinetics parameter calculation

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ABSTRACT

The 2-D/1D whole-core transport method has been widely studied for 3-D fine flux or power distributions and implemented in many deterministic codes. KYCORE, a 2-D/1-D transport code, developed a new iteration strategy to calculate forward flux. In order to perform kinetics parameter computation and sensitivity analysis, an adjoint flux solution is required in modern reactor physics analysis. In this study, a new adjoint neutron transport solver, named KYADJ, was developed by utilizing the 2-D MOC and 1-D SN coupling method. A 3×3 lattice test problem and C5G7 OECD/NEA 3-D benchmarks were used to verify the forward-adjoint neutron transport calculation of KYADJ. Forward-adjoint multiplication factors, forward flux, adjoint flux and kinetics parameters were compared with reference results generated by the Reactor Monte Carlo code RMC. Results showed that KYADJ agreed well with RMC and KYADJ had the ability to provide fine pin-by-pin forward and adjoint flux distributions and accurately compute kinetics parameters.

1. Introduction

An adjoint flux distribution physically represents neutron importance that measures the contribution of neutrons to the steady power in a critical reactor core (Cacuci, 2010). It has two aspects of applications: the adjoint sensitivity analysis based on the perturbation theory Cacuci, 2003) and the quasi-static method in point kinetics equations (Stacey, 2007). The point kinetics equation solutions require the accurate calculation of point reactor kinetics parameters which use the adjoint flux as their weight function. Therefore, the adjoint neutron transport equation has been extensively studied in many advanced codes (Han et al., 2015; Peng et al., 2017; Pusa, 2012; Shokueifar et al., 2016; Zhu et al., 2015a,b).

The “two-step” approach dominates the conventional industrial application and reactor analysis due to its mature technology and convenience. Firstly, a lattice code provides homogenized parameters such as few-group cross-sections and discontinuous factors by conducting the transport calculation of an isolated assembly. Secondly, nodal codes apply these parameters to the whole-core diffusion calculation. The accuracy of this approach remains a serious problem as a result of approximations in models and numerical methods. Compared with the conventional “two-step” approach, the whole-core transport calculation can provide more precise physical quantities at the expense of time and memory (Yuk and Cho, 2015). At the present stage, three-dimensional (3-D) method of characteristics (MOC) encounters great obstacles due to the huge computational overhead. Recent studies have focused on the two-dimensional (2-D)/one-dimensional (1-D) coupled transport calculation (Joo et al., 2004; Tang et al., 2017; Yuk and Cho, 2015; Zhu et al., 2015a,b). Generally, 2-D MOC is used in the radial direction where most inhomogeneity occurs, and different varieties of 1-D difference methods such as diffusion and SN are adopted in the axial direction (Zhu et al., 2015a,b).

The 2-D adjoint transport solutions have been investigated by many researchers in lattice codes (Han et al., 2015; Pusa, 2012; Shokueifar et al., 2016). The MOC and 3-D coarse mesh finite difference (CMFD) adjoint methods have been achieved in MPACT (Zhu et al., 2015a,b). However, few researchers attempt to solve the 2-D/1-D coupled adjoint transport equations. It would be of interest to figure out the fine distribution of 3-D pin-by-pin adjoint flux. We aimed, therefore, to apply the radial MOC and axial SN to solve the forward-adjoint Boltzmann equation, and then verify the validity via kinetics parameter solutions.

KYCORE (Tang et al., 2017), developed by Nuclear Power Institute of China, is a modern 2-D/1-D whole-core transport code designed for the direct transport calculation without any assembly homogenization steps. KYCORE uses the 2-D MOC and 1-D SN coupled by the radial and axial leakage and accelerates the iteration convergence with 3-D CMFD. An optimized procedure has been explored in KYCORE to decrease the computational time and stabilize the convergence of the iteration (Tang

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et al., 2017). On the basis of KYCORE, we have developed a new code, called KYADJ, to perform the forward-adjoint neutron transport calculation. The paper mainly clarifies the theory and method of the forward-adjoint neutron transport in detail, and then shows numerical results for a 3 × 3 lattice test problem and the C5G7 OECD/NEA 3-D benchmarks. Eventually, the paper compares the point reactor kinetics parameters with those calculated from the Reactor Monte Carlo (RMC) code (Wang et al., 2015) for verification.

2. The forward-adjoint neutron transport solutions

In KYADJ, the steady 3-D neutron forward transport equations are solved by the angular direction and energy discretization assuming isotropic scattering. In the cylindrical coordinate system, the equations can be written as

$$\xi_m \frac{\partial \psi_g(\mathbf{r},\mathbf{\Omega}_m)}{\partial r} + \eta_m\left(\frac{\partial \psi_g(\mathbf{r},\mathbf{\Omega}_m)}{r\partial\theta} - \frac{\partial \psi_g(\mathbf{r},\mathbf{\Omega}_m)}{r\partial\omega}\right) + \mu_m \frac{\partial \psi_g(\mathbf{r},\mathbf{\Omega}_m)}{\partial z} + \Sigma_{t,g}(\mathbf{r})\psi_g(\mathbf{r},\mathbf{\Omega}_m) = \frac{1}{4\pi}\sum_{g'=1}^{G}\Sigma_{s,g'\to g}\phi_{g'}(\mathbf{r}) + \frac{1}{4\pi k_{eff}}\chi_g\sum_{g'=1}^{G}\upsilon\Sigma_{f,g'}\phi_{g'}(\mathbf{r}) \tag{1}$$

where $\mathbf{\Omega}_m$ refers to the discrete angular direction, ω denotes the angle between the projection of $\mathbf{\Omega}_m$ on the plane ($\mathbf{e}_r$, $\mathbf{e}_\theta$) and $\mathbf{e}_r$, $\xi_m = \mathbf{\Omega}_m\cdot\mathbf{e}_r$, $\eta_m = \mathbf{\Omega}_m\cdot\mathbf{e}_\theta$, $\mu_m = \mathbf{\Omega}_m\cdot\mathbf{e}_z$, $\mathbf{r}$ is the spatial position, g is the energy group index, ψ_g is the angular flux, k_{eff} is the effective multiplication factor, $\Sigma_{t,g}$ Σ_s, χ_g, υ and $\Sigma_{f,g'}$ are the total cross section, the scattering cross section, the fission neutron energy spectrum, the average number of fission neutrons and the fission cross section, respectively. $\phi_{g'}$ represents the scalar flux approximated by quadrature formula:

$$\phi_g(\mathbf{r}) = \sum_m \omega_m \psi_g(\mathbf{r},\mathbf{\Omega}_m), \tag{2}$$

where ω_m is the quadrature weight.

The corresponding steady 3-D neutron adjoint transport equations can be shown as

$$-\xi_m \frac{\partial \psi^*_g(\mathbf{r},\mathbf{\Omega}_m)}{\partial r} - \eta_m\left(\frac{\partial \psi^*_g(\mathbf{r},\mathbf{\Omega}_m)}{r\partial\theta} - \frac{\partial \psi^*_g(\mathbf{r},\mathbf{\Omega}_m)}{r\partial\omega}\right) - \mu_m \frac{\partial \psi^*_g(\mathbf{r},\mathbf{\Omega}_m)}{\partial z} + \Sigma_{t,g}(\mathbf{r})\psi^*_g(\mathbf{r},\mathbf{\Omega}_m) = \frac{1}{4\pi}\sum_{g'=1}^{G}\Sigma_{s,g\to g'}\phi^*_{g'}(\mathbf{r}) + \frac{1}{4\pi k^*_{eff}}\upsilon\Sigma_{f,g}\sum_{g'=1}^{G}\chi_{g'}\phi^*_{g'}(\mathbf{r}) \tag{3}$$

where ψ^*_g and ϕ^*_g are the adjoint angular flux and adjoint scalar flux, and k^*_{eff} is the effective multiplication factor.

The procedure of the forward neutron transport equation solution has been clarified exhaustively in the previous paper (Tang et al., 2017). Hence we focus on the solving of the adjoint neutron transport equation, especially its differences from the forward neutron transport equation in this paper.

2.1. 2-D MOC adjoint equations

Suppose all cross sections remain unchanged within the axial region $[z_{l-1/2}, z_{l+1/2}]$. Integrating both sides of Eq. (1) over the axial coordinate z and dividing the axial mesh span Δz_l, a serial of radial 2-D forward equations can be obtained:

$$\xi_m \frac{\partial \psi_{g,m,l}(r)}{\partial r} + \Sigma_{t,g}\psi_{g,m,l}(r) = Q_{g,m,l}(r) - L^{Axial}_{g,m,l}(r), \tag{4}$$

where $Q_{g,m,l}$ and $L^{Axial}_{g,m,l}$ are defined as the total source term and the axial leakage term, respectively. Note that only the change of flux along the r direction is considered. The differential terms $\frac{\partial \psi_g(\mathbf{r},\mathbf{\Omega}_m)}{r\partial\theta}$ and $\frac{\partial \psi_g(\mathbf{r},\mathbf{\Omega}_m)}{r\partial\omega}$ are ignored in the 2-D MOC. The total source term and the axial leakage term are expressed as

$$Q_{g,m,l}(r) \equiv \frac{1}{4\pi}\sum_{g'=1}^{G}\Sigma_{s,g'\to g}\phi_{g',l}(\mathbf{r}) + \frac{1}{4\pi k_{eff}}\chi_g\sum_{g'=1}^{G}\upsilon\Sigma_{f,g'}\phi_{g',l}(\mathbf{r}), \tag{5}$$

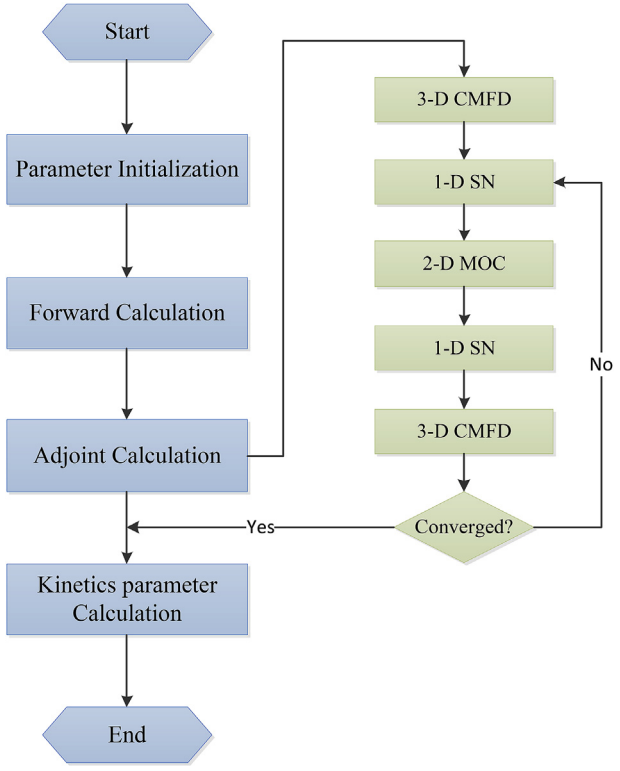

Fig. 1. The code modules and efficient iteration strategy for forward-adjoint calculation.

$$L^{Axial}_{g,m,l}(r) \equiv \frac{\mu_m}{\Delta z_l}\left[\psi_{g,m,l+1/2}(r) - \psi_{g,m,l-1/2}(r)\right]. \tag{6}$$

Once $Q_{g,m,l}$ and $L^{Axial}_{g,m,l}$ are provided, Eq. (4) can be solved by the 2-D MOC which is discussed in detail in Chai's paper (Chai et al., 2016).

The corresponding 2-D MOC adjoint equations can be derived:

$$-\xi_m \frac{\partial \psi^*_{g,m,l}(r)}{\partial r} + \Sigma_{t,g}\psi^*_{g,m,l}(r) = Q^*_{g,m,l}(r) - L^{*Axial}_{g,m,l}(r), \tag{7}$$

where $Q^*_{g,m,l}$ and $L^{*Axial}_{g,m,l}$ are defined as the total adjoint source term and the axial adjoint leakage term is expressed as

$$Q^*_{g,m,l}(r) \equiv \frac{1}{4\pi}\sum_{g'=1}^{G}\Sigma_{s,g\to g'}\phi^*_{g'}(\mathbf{r}) + \frac{1}{4\pi k^*_{eff}}\upsilon\Sigma_{f,g}\sum_{g'=1}^{G}\chi_{g'}\phi^*_{g'}(\mathbf{r}), \tag{8}$$

$$L^{*Axial}_{g,m,l}(r) \equiv -\frac{\mu_m}{\Delta z_l}\left[\psi^*_{g,m,l+1/2}(r) - \psi^*_{g,m,l-1/2}(r)\right]. \tag{9}$$

In comparison with the 2-D forward equations, the adjoint equations exchange $\upsilon\Sigma_{f,g}$ with χ_g and transpose the P0 scattering matrix. The first term in Eq. (7) and the leakage term have the inverse direction with those in Eq. (4). In other words, if the total source term $Q_{g,m,l}$ is replaced merely by the adjoint source term $Q^*_{g,m,l}$ in Eq. (4), the 2-D MOC solver can offer the adjoint angular flux $\psi^*_{g,m',l}$, where the subscript m' signifies the inverse direction with m.

2.2. 1-D SN adjoint equations

In a similar way, integrate both sides of Eq. (1) over the radial mesh p and divide the area Δs. A serial of radial 1-D SN forward equations can be obtained (Tang et al., 2017):

$$\mu_m \frac{\partial \psi_{g,m,p}(z)}{\partial z} + \Sigma_{t,g}\psi_{g,m,p}(z) = Q_{g,m,p}(z) - L^{Radial}_{g,m,p}(z), \tag{10}$$

where $Q_{g,m,p}$ is the total source term which has the same format as Eq. (5). $L^{Radial}_{g,m,p}$ is the radial leakage provided by the 2-D MOC sweep that