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Mathematical treatment for two-point reactor kinetics model of reflected systems

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ABSTRACT

An appropriate precise mathematical model of physical systems has to be developed to predict the dynamic behavior of reflected systems, which comprise core and reflector regions. The present paper developed an accurate and economic mathematical method based on the fundamental matrix to describe the spectrum behavior for one- and two-point kinetics model with multi-group of delayed neutrons. The developed method utilizes the eigenvalues and eigenvectors of the coefficient matrix for the homogeneous linear differential equations resulting from the stiff system of coupled partial differential equations in two-point kinetics model. Moreover, the inverse of the fundamental matrix is calculated analytically. It was evident that the fundamental matrix method is proven to be an excellent solution for cases in which the reactivity is represented by a series of steps and improves accuracy for more general cases of time varying reactivity including Newtonian temperature feedback. The stability of the system is analyzed for different types of reactivity. Finally, the numerical results obtained with these algorithms are applied and verified for different applications of reflected reactors.

1. Introduction

The study of the neutron density is a challenging mathematical and physical problem since it constitutes a fundamental topic in the design and safety of nuclear reactors. An extensive knowledge of the spatial power distribution is required for the design and analysis of different types of current-generation reactors, and that requires the development of more sophisticated theoretical methods. Reflected reactor is one of the most important types of nuclear reactors. Since, the experimental results show that the reactor kinetics behavior of some types of reflector systems cannot be adequately characterized using the point kinetics model. In addition, the neutron lifetimes using theories based on the standard point kinetics model have not always agreed well with the neutron lifetimes predicted by the numerical solutions of the multi-group, multi-dimensional diffusion or transport equations. In view of that, a strongly reflected reactor must be analyzed using two-point reactor kinetics model.

The relatively simple two-point kinetics model has been adopted by Cohn (1962); van Dam (1996). Spriggs et al. (1997) have presented a simplified methodology for determining the coupling parameters and the neutron lifetimes based on simple probability relationships, which describe the aggregate migration of neutrons between core and reflectors. When the coupled kinetics systems have constant coefficients, analytical solutions are easily established, Aboanber and El Mhlawy (2008, 2009), but they are elusive when the coefficients vary with time. In light of Spriggs' two-region kinetics model, a two-group point reactor kinetics model is developed for prompt time constants of a reflected reactor, Tao et al. (2006). Recently, Aboanber and El Mhlawy (2008, 2009) have developed the analytical inversion method (AIM) to solve the derived new version of the reflected core inhour equation that distinct in arrangement from the analogous traditional equation. Furthermore, Aboanber (2009) discussed the effect of fuel temperature feedback for two-point kinetics reactor model due to increasing neutron density in the reactor core. The two-point kinetics model for two energy groups of two-point reactor model of reflected reactors has been developed by Aboanber (2010). The non-linear sliding mode observer which has the robust characteristics facing the parameter uncertainties and disturbances is proposed by Ansarifar et al. (2015), based on the two-point nuclear reactor model to estimate the xenon concentration and delayed neutron precursor density of the Pressurized-Water Nuclear Reactor (PWR) using reactor power measurement.

The present work aims to introduce an effective mathematical method to solve the coupled two-point kinetics equations with multi-group of delayed neutrons for reflected reactors. The developed method utilizes the fundamental matrix of homogenous differential equation for the matrix of two-point kinetics model. The two-point kinetics equations of multi-group of delayed neutrons are introduced in section 2. The fundamental matrix of the homogenous differential equations is

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applied, in section 3, to solve the matrix formula of two-point kinetics model, where the inverse of the fundamental matrix is calculated analytically. In section 4, the numerical results of the proposed method are discussed and compared to the traditional methods. Finally, the general conclusion is presented.

2. Two-point kinetics equations

The two-point reactor kinetics equations of multi-group delayed neutrons are a system of stiff coupled differential equations. The system includes lumped variables and parameters for the core and reflector regions. The two-point kinetics equations can be stated as (van Dam, 1996; Spriggs et al., 1997; Aboanber and El Mhlawy, 2008, 2009; Aboanber, 2009, 2010):

$$\frac{dN_c(t)}{dt} = [\nu\Sigma_f(1-\beta) - \Sigma_{ac} - \Sigma_{cr}]\mathrm{v}N_c(t) + \Sigma_{rc}\mathrm{v}N_r(t) + \sum_{i=1}^{I}\lambda_i C_i(t) + Q_c \tag{1}$$

$$\frac{dN_r(t)}{dt} = \Sigma_{cr}\mathrm{v}N_c(t) - [\Sigma_{ar} + \Sigma_{lr} + \Sigma_{rc}]\mathrm{v}N_r(t) + Q_r \tag{2}$$

$$\frac{dC_i(t)}{dt} = \beta_i\nu\Sigma_f\mathrm{v}N_c(t) - \lambda_i C_i(t), \quad i = 1,2,3,\ \cdots, I \tag{3}$$

where, $N_c(t)$ and $N_r(t)$ are the neutron density in core and reflector regions, $C_i(t)$ is the i-group delayed neutron precursors concentration, Q_c and Q_r are the external neutron source in the core and reflector regions, v is the neutron velocity, ν is the average number of neutrons produced per fission, Σ_f is the macroscopic fission cross section, Σ_{ac} and Σ_{ar} is the macroscopic absorption cross section in core and reflector regions, Σ_{cr} is the macroscopic cross section for neutron transfer from core to reflector, Σ_{rc} is the macroscopic cross section for neutron transfer from reflector to core, Σ_{lr} is the macroscopic leakage cross section for the reflector, λ_i is the decay constant of i-group of delayed neutrons, β_i is the fraction of i-group delayed neutrons and I is the total number of delayed neutron groups.

The interpretation of the individual terms in the ordinary differential equations (1)–(3) are as follows: $\frac{dN_c(t)}{dt}$ and $\frac{dN_r(t)}{dt}$ represent the time rate of change of the neutron density in the core and reflector regions, $\frac{dC_i(t)}{dt}$ is the time rate of change of the i-group delayed neutron precursors concentration, $\nu\Sigma_f(1-\beta)\mathrm{v}N_c(t)$ is the fraction of the prompt neutron density to the fission neutron density after fission process in core region, $\Sigma_{ac}\mathrm{v}N_c(t)$ is the absorption neutron density in the core, $\Sigma_{cr}\mathrm{v}N_c(t)$ is the neutron current from core to reflector, $\Sigma_{rc}\mathrm{v}N_r(t)$ is the neutron current from reflector to core, $\sum_{i=1}^{I}\lambda_i C_i(t)$ represents the delayed neutrons which appear in the core region due to decay of the precursor concentrations, $\Sigma_{ar}\mathrm{v}N_r(t)$ is the absorption neutron density in the reflector region, $\Sigma_{lr}\mathrm{v}N_r(t)$ is the leakage neutron current from the reflector region to outside and $\beta_i\nu\Sigma_f\mathrm{v}N_c(t)$ is the i-group of the delayed neutron precursors in the core region, Fig. 1.

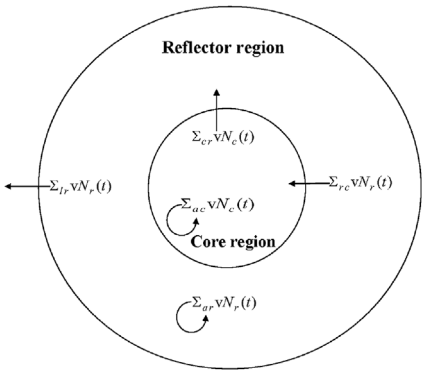

Fig. 1. Relation between the core and reflector regions.

Equations (1)–(3) are simplified as:

$$\frac{dN_c(t)}{dt} = \frac{\rho - \beta - \alpha_{cr}\alpha_{rc}}{\Lambda_c}N_c(t) + \frac{\alpha_{rc}}{\Lambda_r}N_r(t) + \sum_{i=1}^{I}\lambda_i C_i(t) + Q_c \tag{4}$$

$$\frac{dN_r(t)}{dt} = \frac{\alpha_{cr}}{\Lambda_c}N_c(t) - \frac{1}{\Lambda_r}N_r(t) + Q_r \tag{5}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda_c}N_c(t) - \lambda_i C_i(t), \quad i = 1,2,3,\ \cdots, I \tag{6}$$

where, $\rho = 1 - \frac{\Sigma_{ac}}{\nu\Sigma_f} - \frac{\Sigma_{cr}(\Sigma_{ar}+\Sigma_{lr})}{\nu\Sigma_f(\Sigma_{ar}+\Sigma_{lr}+\Sigma_{rc})}$ is the reactivity in core, $\Lambda_c = \frac{1}{\mathrm{v}\nu\Sigma_f}$ is the neutron generation time in core, $\Lambda_r = \frac{1}{\mathrm{v}(\Sigma_{ar}+\Sigma_{lr}+\Sigma_{rc})}$ is the neutron lifetime in the reflector, $\alpha_{cr} = \frac{\Sigma_{cr}}{\nu\Sigma_f}$ is the fraction of fission neutrons produced that escapes to the reflector, $\alpha_{rc} = \frac{\Sigma_{rc}}{\Sigma_{ar}+\Sigma_{lr}+\Sigma_{rc}}$ is the fraction of reflector neutrons flowing to the core (the reflection coefficient or albedo) and the initial conditions are as follows: $N_c(0) = 1.0(neutron/cm^3)$, $N_r(0) = \frac{\alpha_{cr}\Lambda_r}{\Lambda_c}N_c(0) + \Lambda_r Q_r$, and $C_i(0) = \frac{\lambda_i\beta_i}{\Lambda_c}N_c(0)$.

Equations (4)–(6) are called *two-point kinetics equations with multi-group delayed neutrons*. The solution of this system of ordinary differential equations is our target in the following section using the fundamental matrix method.

3. Fundamental matrix

Accurate values for the model parameters should be obtained from detailed transport calculations on the system. To suit our case the set of the two-point kinetics model with multi-group of delayed neutrons may be written in matrix notation as:

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{A}\mathbf{X}(t) + \mathbf{B} \tag{7}$$

where,

$$\mathbf{X}(t) = \begin{bmatrix} N_c(t) \\ N_r(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_I(t) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} Q_c \\ Q_r \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \alpha_1 & \alpha_2 & \lambda_1 & \lambda_2 & \cdots & \lambda_I \\ \xi_1 & -\xi_2 & 0 & 0 & \cdots & 0 \\ \mu_1 & 0 & -\lambda_1 & 0 & \cdots & 0 \\ \mu_2 & 0 & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mu_I & 0 & 0 & 0 & \cdots & -\lambda_I \end{bmatrix},$$

and $\alpha_1 = \frac{\rho-\beta-\alpha_{cr}\alpha_{rc}}{\Lambda_c}$, $\alpha_2 = \frac{\alpha_{rc}}{\Lambda_r}$, $\xi_1 = \frac{\alpha_{cr}}{\Lambda_c}$, $\xi_2 = \frac{1}{\Lambda_r}$, and $\mu_i = \frac{\beta_i}{\Lambda_c}$.

The solution of equation (7) depends on the coefficient matrix **A** for the following two cases:

Case I: The matrix **A** is time independent, analytical solution e.g. (Aboanber and Nahla, 2002; Aboanber, 2003a, b; Jordan and Smith, 2007; Spriggs et al., 1997), of equation (7) is easily obtained:

$$\mathbf{X}(t) = \Phi(t)\Phi^{-1}(t_0)\mathbf{X}(t_0) + \Phi(t)\int_{t_0}^{t}\Phi^{-1}(s)\mathbf{B}ds \tag{8}$$

where, $\Phi(t)$ is a fundamental matrix defined as follows

$$\Phi(t) = \left[\mathbf{U}_0 e^{\omega_0 t} \quad \mathbf{U}_1 e^{\omega_1 t} \quad \mathbf{U}_2 e^{\omega_2 t} \quad \cdots \quad \mathbf{U}_{I+1} e^{\omega_{I+1} t}\right] \tag{9}$$

ω_k and $\mathbf{U}_k$ are the eigenvalues and eigenvectors of the matrix **A** respectively, $k = 0,1,2,\ \cdots, I+1$.

Case II: The matrix **A** is time dependent (Aboanber, 2009), the solution of equation (7) over time interval $[t_m,\ t_{m+1}]$ can be obtained in the following procedure:

$$\mathbf{X}(t_{m+1}) = \Phi(t_{m+1})\Phi^{-1}(t_m)\mathbf{X}(t_m) + \Phi(t_{m+1})\int_{t_m}^{t_{m+1}}\Phi^{-1}(s)\mathbf{B}ds \tag{10}$$

Here $\Phi(t_{m+1})$ is the fundamental matrix at t_{m+1} which takes the mode: