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Mittag-Leffler and Padé approximations to stiff fractional two point kinetics equations

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ABSTRACT

The mathematical model of stiff two-point kinetics equations for a compact core with bulky reflector plays a basic role in kinetics schemes of reflector reactor in the nuclear reactor dynamic systems. This model is a non-conventional models describe the number of neutrons per volume and delayed precursor into reactors. To analyze reactor response, a novel mathematical treatment should be introduced to solve the stiff fractional differential equations in the matrix form. Mittag-Leffler functions have recently caught the interest, particularly with the models which using fractional calculus. In this work, a fractional formula of the coupled stiff two-point reactor kinetics model with I-groups of delayed neutrons are considered by merging Mittag-Leffler function with the developed Padé approximations. The validity of the proposed fractional formula is confirmed for various reactivity such as step and ramp reactivity in various fractional order. The estimated values confirm that the treatment for the problem under consideration using Mittag-Leffler function and the developed Padé approximations agree with Picard iteration method and the conformist techniques.

1. Introduction

The important fact in the nuclear reactor is that the fuel is a very expensive commodity, a large number of developmental research are introduced to reduce the cost of the reactors. One way of reducing the cost of the reactors is to provide reflectors. The neutron reflector reflects the most escape neutrons to the reactor core i.e. they reduce neutron leakage. The neutron reflector reflects the most escape neutrons to the reactor core i.e. they reduce neutron leakage. Practically, design of the most power reactors reduced the neutron leakage except in the case of research reactors. As a result of the reduction of neutron leakage, the neutron reflectors increases the effective multiplication factors and reduces the amount of fuel needed to maintain the efficiency of power reactor for a long time. Furthermore, the reflected neutrons to the core region increases chain reaction. It is obvious that, in the case of sustained the core size, the reflector introduces additional reactivity which is important for higher fuel burnup. For these reasons, a strongly reflected reactor represented by two-point kinetic system is one of the significant classes of nuclear power reactors.

Numerous theories and models have been developed for the solution of the coupled system of reflected reactors (Avery et al., 1958; Wasserman, 1960; Cohn, 1962; Kondor and Kosaly, 1971; Shinkawa et al., 1978; Yamane et al., 1980; Nishina and Yamane, 1985; Kobayashi, 1990; Dam, 1996; Spriggs et al., 1997; Aboanber, 2009, 2010; Aboanber and El-Mhlawy, 2008, 2009; Ansarifar et al., 2015). In the framework of reactor kinetics field, Aboanber (2003a, b) presented a general frame formula for the coefficients of the inhour equation which depends on the inserted reactivity and delayed neutron parameters. In determining the coefficients of such a polynomial, unusual scientific mathematical formulation is introduced for the period-reactivity equation for I groups of delayed neutrons and for D_2O-and Be-moderated reactors. This mathematical frame is a polynomial with $(I + 1)$ degree for I^{th} groups of delayed neutrons. Likewise, the coefficients of such polynomial are derived and represented in a general formula. These coefficients have a direct reliance on various sort of inserted reactivity.

Recently, the scientists used fractional calculus to describe the physical phenomena, it is more realistic and more popular. In addition, the fractional models plays an outstanding role from the viewpoint of applications in modeling of many reaction processes, bio-mathematical, economics and many problems in chemistry, physics and engineering. One of their applications is the nuclear reactor dynamics model. The importance of this subject, which describes the neutron density and the delayed precursor concentration into nuclear reactors, is the topic of incalculable examining and applications to understand the effect of neutron dynamics behavior. The neutron density plays an important

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Table 1
Types of Padé approximations.

Type	Explicit form	Coefficients	
$P_{0,1}$	$\frac{I}{I+b(\mathbf{A}t^{\kappa})}$		$b=\frac{-1}{\Gamma(1+\kappa)}$
$P_{1,1}$	$\frac{I+a(\mathbf{A}t^{\kappa})}{I+b(\mathbf{A}t^{\kappa})}$	$a=\frac{1}{\Gamma(1+\kappa)-\frac{\Gamma(1+\kappa)}{\Gamma(1+2\kappa)}}$	$b=\frac{-\Gamma(1+\kappa)}{\Gamma(1+2\kappa)}$
$P_{0,2}$	$\frac{I}{I+b_1(\mathbf{A}t^{\kappa})+b_2(\mathbf{A}t^{\kappa})^2}$		$b_1=\frac{-1}{\Gamma(1+\kappa)}$
			$b_2=\frac{1}{\Gamma(1+\kappa)^2-\frac{1}{\Gamma(1+2\kappa)}}$
$P_{1,2}$	$\frac{I+a_1(\mathbf{A}t^{\kappa})}{I+b_1(\mathbf{A}t^{\kappa})+b_2(\mathbf{A}t^{\kappa})^2}$	$a_1=\frac{(2\Gamma(1+\kappa)^2+\Gamma(1+2\kappa))}{(\Gamma(1+\kappa)^3+\Gamma(1+\kappa)\Gamma(1+2\kappa))}$	$b_1=\frac{-1}{\Gamma(1+\kappa)}$
			$b_2=\frac{1}{\Gamma(1+\kappa)^2-\frac{1}{\Gamma(1+2\kappa)}}$
$P_{2,2}$	$\frac{I+a_1(\mathbf{A}t^{\kappa})+a_2(\mathbf{A}t^{\kappa})^2}{I+b_1(\mathbf{A}t^{\kappa})+b_2(\mathbf{A}t^{\kappa})^2}$	$a_1=b_1+\frac{1}{\Gamma(1+\kappa)}$	$b_1=-\frac{\Gamma(1+3\kappa)}{\Gamma(1+4\kappa)}-b_2\frac{\Gamma(1+3\kappa)}{\Gamma(1+2\kappa)}$
		$a_2=\frac{1}{\Gamma(1+2\kappa)}+b_2+b_1\frac{1}{\Gamma(1+\kappa)}$	$b_2=\frac{\left(\frac{\Gamma(1+\kappa)\Gamma(1+3\kappa)}{\Gamma(1+2\kappa)\Gamma(1+4\kappa)}\right)-\left(\frac{\Gamma(1+\kappa)}{\Gamma(1+3\kappa)}\right)}{\left(1-\frac{\Gamma(1+\kappa)\Gamma(1+3\kappa)}{\Gamma(1+2\kappa)^2}\right)}$

where, **I** is the unit matrix.

Table 2
Neutron density for step reactivity insertion.

t	Padé11 Approximation			Mittag-Leffler			Mittag-Padé		
(s)	$\kappa=1.02$	$\kappa=1.0$	$\kappa=0.98$	$\kappa=1.02$	$\kappa=1.0$	$\kappa=0.98$	$\kappa=1.02$	$\kappa=1.0$	$\kappa=0.98$
Core									
0.0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
1.0	1.9265200	1.9863688	2.0493630	1.9265150	1.9863632	2.0493569	1.9265150	1.9863632	2.0493569
2.0	2.4267376	2.5245366	2.6322449	2.4267360	2.5245350	2.6322433	2.4267360	2.5245350	2.6322433
3.0	2.8911496	3.0436072	3.2164399	2.8911489	3.0436065	3.2164391	2.8911489	3.0436065	3.2164391
5.0	3.9304553	4.2447387	4.6145417	3.9304548	4.2447380	4.6145408	3.9304548	4.2447380	4.6145408
Reflector									
0.0	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000
1.0	15.3916051	15.8713808	16.3761931	15.3915647	15.8713359	16.3761441	15.3915647	15.8713359	16.3761441
2.0	19.3975063	20.1799882	21.0416217	19.3974934	20.1799752	21.0416087	19.3974934	20.1799752	21.0416087
3.0	23.1124208	24.3316774	25.7138031	23.1124153	24.3316715	25.7137966	23.1124153	24.3316715	25.7137966
5.0	31.4232815	33.9362779	36.8931668	31.4232776	33.9362727	36.8931596	31.4232776	33.9362727	36.8931596

Table 3
Neutron density for ramp reactivity insertion $\rho=0.1(\beta t)$.

t	Padé11 Approximation			Mittag-Leffler			Mittag-Padé		
(s)	$\kappa=1.02$	$\kappa=1.0$	$\kappa=0.98$	$\kappa=1.02$	$\kappa=1.0$	$\kappa=0.98$	$\kappa=1.02$	$\kappa=1.0$	$\kappa=0.98$
Core									
0.0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
1.0	1.0795965	1.0834706	1.0873576	1.0795892	1.0834630	1.0873496	1.0795892	1.0834630	1.0873496
2.0	1.2320511	1.2421136	1.2524520	1.2320427	1.2421047	1.2524426	1.2320427	1.2421047	1.2524426
3.0	1.4587015	1.4806447	1.5037590	1.4586916	1.4806342	1.5037479	1.4586916	1.4806342	1.5037479
5.0	2.3334026	2.4260705	2.5282196	2.3333875	2.4260542	2.5282019	2.3333875	2.4260542	2.5282019
Reflector									
0.0	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000	8.0000000
1.0	8.6324598	8.6637173	8.6950652	8.6324539	8.6637086	8.6950532	8.6324539	8.6637086	8.6950532
2.0	9.8498714	9.9307296	10.0137706	9.8498628	9.9307180	10.0137555	9.8498628	9.9307180	10.0137555
3.0	11.6599054	11.8358589	12.0211501	11.6598962	11.8358464	12.0211335	11.6598962	11.8358464	12.0211335
5.0	18.6434859	19.3849605	20.2021989	18.6434773	19.3849471	20.2021794	18.6434773	19.3849471	20.2021794

role for calculating power and safety of nuclear reactors, which is an interesting subject for nuclear reactor scientists to adapt and stratify mathematical method such as the fractional calculus for the two-point kinetics equations (FNTPKE). The first scientific group derived the FNPKE is Espinosa-Paredes et al. (2011) and Polo-Labarrios et al. (2014). Then, Ray and Patra (2014) presented numerical imitation to solve another fractional model of point kinetic equations via the multi-step differential transform method. Aboanber and Nahla (2016, 2017) developed the analytical approximation for the modified formula of the fractional point kinetics equations with and without multigroup of delayed neutrons during start-up of a nuclear reactor. The main goal of this work is to study the fractional model for two point kinetics equations using different approximation techniques such as Picard iterative method, Padé01 (Backward Euler), and Padé11 (Crank-Nicholson) approximations. Whereas, Picard iteration represents a productive numerical method for determining the existence and uniqueness of