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The multi-physics improved quasi-static method - Application to a neutronics-thermomechanics coupling

Cyril Patricot [a, *], Anne-Marie Baudron [a], Olivier Fandeur [b]

[a] CEA, DEN, DM2S, SERMA, F-91191 Gif-sur-Yvette, France
[b] CEA, DEN, DM2S, SEMT, F-91191 Gif-sur-Yvette, France

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ABSTRACT

The quasi-static method is widely used for space- and time-dependent neutron transport problems. It is based on the factorization of the flux into the product of two functions, an "amplitude" depending only on time and a "shape" which depends on all variables. Thanks to this factorization, long time-steps can be used for the computation of the shape, leading to a substantial reduction of the calculation time. Two algorithms, based on the quasi-static factorization, can be found in the literature: the "Improved Quasi-static Method" (IQM), and the "Predictor-Corrector Quasi-static Method" (PCQM).

In this paper we show, on the example of the Godiva experiment, that the IQM algorithm can be easily adapted to multi-physics simulations. Moreover, most of the common coupling or time-step control strategies are compatible with this algorithm and we test some of them here. In particular, a technique taken from existing codes with point-kinetic modules and based on feedback coefficients is found, in our case, to be especially efficient and gives precise and fast results. This shows that the multi-physics IQM presented in this paper is compatible with these existing codes, and may be a way to couple them with neutron transport solvers.

1. Introduction

The quasi-static approach (introduced more than fifty years ago by Allan Henry (Henry, 1958; Henry and Curlee, 1958)) is widely used for solving space- and time-dependent neutron transport problems. It is based on the factorization of the neutron flux into an amplitude and a shape functions:

$$\phi(\boldsymbol{r}, \boldsymbol{\Omega},\ E,\ t) = N(t)\psi(\boldsymbol{r}, \boldsymbol{\Omega}, E, t). \tag{1}$$

We introduce the diagonal "inertial operator" $I_{1/V}$ multiplying the flux by the inverse of the neutron speed. $<.,.>$ denotes the classic scalar product. With these notations, the factorization (1) is defined by a constraint on the shape:

$$\frac{\partial}{\partial t}\left\langle I_{1/V}\psi, f\right\rangle = 0, \tag{2}$$

where f is an arbitrary weighting function. The steady-state adjoint flux ϕ_0^* is usually chosen as weighting function, and is used in this work.

There are two well-known ways of implementing the factorization (1). The historical approach (Ott and Meneley, 1969; Dahmani, 1999) is called "Improved Quasi-static Method" (IQM) and is based on an amplitude (N in (1)) computation followed by a shape (ψ in (1)) one. This procedure is repeated until a convergence criterion on the constraint (2) is fulfilled. A more recent approach (Dulla et al., 2008), called "Predictor-Corrector Quasi-static Method" (PCQM) is based on a flux (ϕ in (1)) computation followed by an amplitude one, used to correct the flux prediction. No iterations are made. Both methods use two different time-scales: a micro one (dt) for the amplitude calculation and a macro one (Δt) for the shape or flux calculation.

The structure of the IQM algorithm seems better suited for solving multi-physics problems, and could lead to efficient coupling strategies. The purpose of this paper is to highlight the potential of IQM for multi-physics calculations, and to discuss some aspects of its use in this frame. We illustrate this idea with an original neutron transport-thermomechanics coupling example: a prompt critical burst of the Godiva experiment (Wimett, 1956), which provides a strong and non-monotonic time-dependent coupling. CAST3M ("CAST3M", 2015), a finite element code dedicated to structural mechanics, is used to compute the transient. In particular, a neutron transport solver, presented in (Patricot et al., 2016) and able to deal with irregular meshes, is used.

* Corresponding author.
E-mail address: cyril.patricot@cea.fr (C. Patricot).

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Section 2 recalls the quasi-static method and its two approaches, IQM and PCQM. Section 3 presents the Godiva experiment and the models used. Sections 4 and 5 discuss the transient computation and compare some coupling strategies with, respectively, fixed and variable time-steps. Finally, section 6 concludes the paper.

2. The quasi-static method

2.1. The quasi-static equations

Consider the time-dependent neutron transport and delayed neutron precursor equations:

$$\begin{cases} I_{1/V}\dfrac{\partial \phi}{\partial t} = \mathscr{L}\phi + F_p\phi + \displaystyle\sum_l \chi_l \lambda_l C_l \\ \dfrac{\partial C_l}{\partial t} = -\lambda_l C_l + \dfrac{1}{\chi_l} F_l \phi, \end{cases} \tag{3}$$

where C_l denotes the normalized concentration of delayed neutron precursors of type l, $\mathscr{L}$ the transport operator including streaming, absorption and scattering, F_p the prompt neutron production rate operator, F_l the delayed neutron production rate operator and $I_{1/V}$ the inertial operator. χ_l and λ_l are the delayed fission spectrum and the decay constant of the precursor of type l.

Taking the scalar product of (3) with the weighting function, and using (2), we obtain the point-kinetic equations satisfied by the amplitude function:

$$\begin{cases} \dfrac{\partial N}{\partial t} = \dfrac{\rho - \overline{\beta}}{\Lambda} N + \displaystyle\sum_l \lambda_l c_l \\ \dfrac{\partial c_l}{\partial t} = -\lambda_l c_l + \dfrac{\overline{\beta_l}}{\Lambda} N. \end{cases} \tag{4}$$

The point-kinetic parameters are ($F = F_p + \sum_l F_l$ is the total neutron production rate operator):

$$\rho = \frac{\langle (\mathscr{L} + F)\psi, \phi_0^* \rangle}{\langle F\psi, \phi_0^* \rangle}, \tag{5}$$

$$\Lambda = \frac{\langle I_{1/V}\psi, \phi_0^* \rangle}{\langle F\psi, \phi_0^* \rangle}, \tag{6}$$

$$\overline{\beta_l} = \frac{\langle F_l\psi, \phi_0^* \rangle}{\langle F\psi, \phi_0^* \rangle}, \tag{7}$$

$$\overline{\beta} = \sum_l \overline{\beta_l}, \tag{8}$$

$$c_l = \frac{\langle \chi_l C_l, \phi_0^* \rangle}{\langle I_{1/V}\psi, \phi_0^* \rangle}. \tag{9}$$

The system of coupled equations satisfied by ψ and C_l is then obtained by replacing the factorization (1) in (3):

$$\begin{cases} I_{1/V}\left(\dfrac{\partial \psi}{\partial t} + \psi \dfrac{1}{N}\dfrac{\partial N}{\partial t}\right) = \mathscr{L}\psi + F_p\psi + \dfrac{1}{N}\displaystyle\sum_l \chi_l \lambda_l C_l \\ \dfrac{\partial C_l}{\partial t} = -\lambda_l C_l + \dfrac{N}{\chi_l} F_l \psi \end{cases} \tag{10}$$

2.2. The two quasi-static approaches

2.2.1. The improved quasi-static method (IQM)

The IQM, introduced first in (Ott and Meneley, 1969), consists in solving the two coupled systems of equations (4) and (10). The algorithm can be summarized as follows (c is a convergence criterion chosen by user):

- Step 1: The point-kinetic equations (4) are solved first, over a macro time interval Δt, using micro time-steps dt.
- Step 2: Knowing the amplitude and its time derivative at $t + \Delta t$, the shape equations (10) are then solved, directly with Δt as a time-step.
- Step 3: The error ε on the normalization (2) is evaluated and the shape is renormalized. This is necessary because the point-kinetic equations assume that the shape satisfies the normalization condition (2).
- Step 4: Point kinetic parameters are computed.
- Step 5: Step 1–4 are iterated until $\varepsilon < c$.

Several numerical schemes are possible on this basis.

2.2.2. The predictor-corrector quasi-static method (PCQM)

In the PCQM (Dulla et al., 2008) the flux equations (3) are coupled with the point-kinetic ones (4). The algorithm can be summarized as follows:

- Step 1: The flux equations (3) are solved first, over a macro time-step Δt.
- Step 2: An approximate shape function is derived at $t + \Delta t$ from the flux of step 1 and condition (2).
- Step 3: Point-kinetic parameters are evaluated and the point-kinetic equations (4) are solved, over the macro time interval Δt using micro time-steps dt.
- Step 4: Flux is reconstructed as the product of the shape of step 2 and of the amplitude of step 3.
- Step 5: Precursor concentrations are recomputed with this new flux.

Here again, details of the implementation can vary.

2.2.3. Block diagrams

Block diagrams for the two algorithms are sketched in Fig. 1:

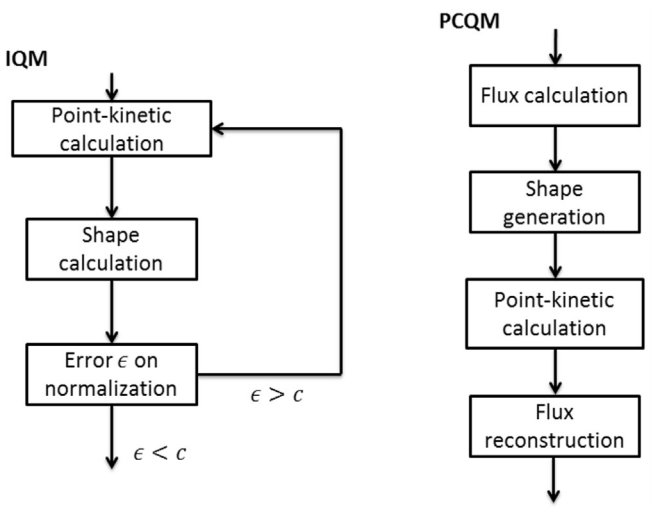

Fig. 1. Block diagrams for IQM and PCQM calculation of a time-step.

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