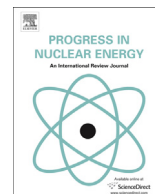




Contents lists available at ScienceDirect

## Progress in Nuclear Energy

journal homepage: [www.elsevier.com/locate/pnucene](http://www.elsevier.com/locate/pnucene)

# Comparison of two three-dimensional heterogeneous Variational Nodal Methods for PWR control rod cusping effect and pin-by-pin calculation

Yongping Wang, Hongchun Wu, Yunzhao Li\*

School of Nuclear Science and Technology, Xi'an Jiaotong University, 28 West Xianning Road, Xi'an, Shaanxi 710049, China

## ARTICLE INFO

## Article history:

Received 11 September 2016

Received in revised form

9 December 2016

Accepted 1 June 2017

Available online xxx

## Keywords:

Variational Nodal Method

Heterogeneous

Pin-by-pin

Control rod cusping effect

## ABSTRACT

Two heterogeneous nodal methods based on the Variational Nodal Method (VNM) are investigated with diffusion approximation in three-dimensional Cartesian geometry. The first one is named as Function Expansion (FE) method while the second is Finite Sub-element (FS) method. Based on our previous work and the code Violet-Het1D in one-dimensional slab geometry, a code named Violet-Het3D was developed to handle the Pressurized Water Reactor (PWR) control rod cusping effect and pin-by-pin calculation by using either of these two methods. To eliminate the control rod cusping effect, Violet-Het3D provides a different idea from the existing methods. Neither homogenization procedure nor mesh adjustment is needed in Violet-Het3D by taking advantage of the treatment for heterogeneous nodes. Numerical results show that both the FE and FS methods can eliminate the cusping effect and obtain accurate power distribution while the FE method has relatively higher efficiency and accuracy. In contrast, for pin-by-pin calculation, the FS method obtains more accurate eigenvalue and pin power distribution than the FE method.

© 2017 Published by Elsevier Ltd.

## 1. Introduction

In recent decades, the two-step scheme (Smith, 1986) is employed in Pressurized Water Reactor (PWR) core computation: lattice calculation with homogenization and whole-core diffusion calculation with pin-power reconstruction. For the core diffusion calculation, nodal methods have been widely employed. In nodal methods, there is one basic assumption that the cross sections within each node are homogeneous. Although the assumption is reasonable in most situations in PWR as the detailed construction within each assembly is homogenized by using the lattice code, it will introduce errors or limit the computational efficiency in some special cases. Two typical examples including the “control rod cusping effect” and pin-by-pin calculation are discussed in this paper.

Firstly, control rods keep moving along the axial direction within the PWR core with a step size of about 1–2 cm, while the nodal size of neutronics simulation is usually about 10–20 cm. Thus, unavoidably a control rod assembly may be partially inserted

into a node which means part of the node uses assembly-homogenized cross sections with control rods in while the other part uses assembly-homogenized cross sections with control rods out. As the piece-wise distributed cross sections within a node is not allowed by traditional nodal methods, the heterogeneous node should be homogenized. However, if we simply homogenize the node by using the volume-weighted scheme, it would result a lot of wiggles in the numerically simulated curve of control rod differential worth which theoretically should be smooth. This phenomenon is the so-called control rod cusping effect (Si, 2006). Since 1980s (Han-Sem, 1984), many methods have been investigated to eliminate it. These methods can be classified into two categories although they have different implementations. The first category is flux-volume-weighted methods (Yamamoto, 2004; Dall'Osso, 2002; Bandini et al., 2003; Downar et al., 2004; Reitsma and Muller, 2002). They have to obtain an approximate flux distribution over the heterogeneous nodes for homogenization. The second category is adaptive mesh methods (Zhang, 2014). They adjust the spatial mesh after each control rod movement to avoid the

\* Corresponding author.

E-mail address: [yunzhao@mail.xjtu.edu.cn](mailto:yunzhao@mail.xjtu.edu.cn) (Y. Li).

appearance of heterogeneous nodes.

Secondly, to reduce the error introduced by assembly homogenization, the pin-by-pin scheme has arisen. It solves the whole-core problem with pin cell homogenized cross sections to eliminate the assembly homogenization and pin-power reconstruction. Therefore, many pin-by-pin calculation codes have been developed such as SCOPE2 (Tatsumi and Yamamoto, 2003) and EFEN (Li et al., 2014a). However, as traditional nodal method requires homogenized cross sections in each mesh, the whole-core pin-by-pin problem should consist of millions of meshes, causing issues in both memory and efficiency. For example, a PWR can be divided into 10 million pin-size meshes. Together with the SP<sub>3</sub> approximation and 4 energy groups, it needs 10 GB memory and the computational time is about 24 h (Yang et al., 2014) for one single CPU.

For the control rod cusping effect problem, if nodal method allows heterogeneity in a node, neither the flux-volume-weighted method nor the mesh adjustment would be needed to avoid the heterogeneous nodes. The control rod cusping effect is supposed to be directly eliminated by using heterogeneous nodal method. For the pin-by-pin problem, a whole assembly can be treated as a node if heterogeneity is allowed in a node.

Thus, to eliminate the requirement of nodal homogeneous cross sections in traditional nodal methods, heterogeneous nodal methods were developed. In 1997, Fanning and Palmiotti (1997) developed a heterogeneous Variational Nodal Method (VNM (Palmiotti et al., 1995; Li et al., 2015a)). The space variable of flux and current in this method are still expanded by polynomials as same as that in homogeneous VNM. For calculating the response matrices, the heterogeneous node is divided into several homogeneous regions and then the integrals over the node are divided into a set of homogeneous integrals. In 2003, Smith et al. (2003) developed another heterogeneous VNM. The main idea of this method is to further break each heterogeneous node into sub-elements within which the cross sections are constants. The flux is then expanded by finite trial functions in space. The nodal functional is constructed by the functional of all sub-elements in the node.

To investigate and make comparisons of the above two methods, this paper derives the formulations based on the two methods with diffusion approximation in three-dimensional Cartesian geometry. In this paper, we call them the function expansion (FE) method (Li et al., 2014b) and finite sub-element (FS) (Li et al., 2015b) method respectively. However, different from Fanning's method, the cross sections in FE method are presented by piecewise polynomials which means it can treat not only the problems with heterogeneous nodes consisted of several homogeneous regions as that in Fanning's paper, but also the problems with continuous cross sections (Li et al., 2014b) within the nodes. Different from Smith's method, the nodal interface current in FS method is expanded by finite trial functions instead of polynomials which should eliminate the interface approximation caused by the transformation of coefficients between finite trial functions and polynomials. Moreover, this paper applies these two methods to eliminating the control rod cusping effect in PWR which is not found in previous works. This might be the first practical application of heterogeneous nodal methods.

A commercial program called Freefem++ (Bernardi et al., ) is employed to generate the sub-elements inside the nodes. A code named Violet-Het3D was developed to treat heterogeneous node by the two methods. In addition, a representative PWR control rod cusping effect problem and a pin-by-pin problem were employed in this paper to make the comparisons for these two methods.

## 2. Theory

Both of the FE and FS methods start from the three-dimensional within-group diffusion equation:

$$\begin{cases} \nabla \cdot \mathbf{J}(\mathbf{r}) + \Sigma_t(\mathbf{r})\Phi(\mathbf{r}) = \Sigma_s(\mathbf{r})\Phi(\mathbf{r}) + S(\mathbf{r}) \\ \frac{1}{3}\nabla\Phi(\mathbf{r}) + \Sigma_t(\mathbf{r})\mathbf{J}(\mathbf{r}) = 0 \end{cases} \quad (1)$$

where  $\Phi$  is the scalar flux ( $\text{cm}^{-2} \cdot \text{s}^{-1}$ ),  $\mathbf{J}$  is net current ( $\text{cm}^{-2} \cdot \text{s}^{-1}$ ),  $\Sigma_t$  is the total cross section ( $\text{cm}^{-1}$ ),  $\Sigma_s$  is the within-group scattering cross section ( $\text{cm}^{-1}$ ), and  $S$  is the source term ( $\text{cm}^{-3} \cdot \text{s}^{-1}$ ) including scattering and fission:

$$S(\mathbf{r}) = \sum (\mathbf{r})\Phi(\mathbf{r}) + \frac{1}{k}Q(\mathbf{r})\Phi(\mathbf{r}) \quad (2)$$

where

$$\sum (\mathbf{r})\Phi(\mathbf{r}) = \sum_{g' \neq g} \Sigma_{gg'}^s(\mathbf{r})\Phi_{g'}(\mathbf{r}) \quad (3)$$

and

$$\frac{1}{k}Q(\mathbf{r})\Phi(\mathbf{r}) = \sum_g \frac{\chi_g}{k} \nu \Sigma_{fg}(\mathbf{r})\Phi_g(\mathbf{r}) \quad (4)$$

$k$  is the effective multiplication factor,  $\chi_g \nu \Sigma_{fg}$  and  $\Sigma_{gg'}^s$  are respectively the fission and scattering cross sections ( $\text{cm}^{-1}$ ) from energy group  $g'$  to  $g$ .

The same as the homogeneous VNM, the entire problem domain is decomposed into subdomains  $V_v$  (nodes) and the functional can be written as a superposition of nodal contributions:

$$F[\Phi, J] = \sum_v F_v[\Phi, J] \quad (5)$$

$F[\Phi, J]$  stands for the functional of the whole problem in terms of  $\Phi$  and  $J$  while  $F_v[\Phi, J]$  is the nodal functional. Start from here, the FE and FS methods treat the nodal heterogeneity differently.

### 2.1. Function expansion method

The nodal functional consists of volume and surface contributions:

$$F_v[\Phi, J] = \int_v dV \left\{ \frac{1}{3} \Sigma_t(\mathbf{r})^{-1} (\nabla \Phi(\mathbf{r}))^2 + (\Sigma_t(\mathbf{r}) - \Sigma_s(\mathbf{r})) \Phi(\mathbf{r})^2 - 2 \Phi(\mathbf{r}) S(\mathbf{r}) \right\} + 2 \sum_\gamma \int_\gamma \Phi_\gamma J_\gamma d\Gamma \quad (6)$$

$$J_\gamma = \mathbf{J}_\gamma \cdot \mathbf{n}_\gamma \quad (7)$$

where  $v$  stands for a certain node of the entire problem;  $\gamma$  represents a certain nodal surface of node  $v$ ;  $J_\gamma$  stands for the net current ( $\text{cm}^{-2} \cdot \text{s}^{-1}$ ) on the surfaces of the node and  $\mathbf{n}_\gamma$  is the outer normal of the nodal surface. In this method, the cross sections in the nodal functional as shown in Eq. (6) can be written as piecewise polynomials instead of constants. Thus, it can treat both the problems of heterogeneous nodes with piecewise homogeneous regions and the problems with continuous cross sections.

Download English Version:

<https://daneshyari.com/en/article/8084611>

Download Persian Version:

<https://daneshyari.com/article/8084611>

[Daneshyari.com](https://daneshyari.com)