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The multilevel quasidiffusion method with multigrid in energy for eigenvalue transport problems

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ABSTRACT

A multilevel iterative method for solving multigroup neutron transport k-eigenvalue problems in two-dimensional geometry is developed. This method is based on a system of group low-order quasidiffusion (LOQD) equations defined on a sequence of coarsening energy grids. The spatial discretization of the LOQD equations uses compensation terms which make it consistent with a high-order transport scheme on a given spatial grid. Different multigrid algorithms are applied to solve the multilevel system of group LOQD equations on grids in energy. The eigenvalue is evaluated from the LOQD problem on a coarsest grid. To further improve the efficiency of iterative schemes hybrid multigrid algorithms are developed. The numerical results of tests with a large number groups are presented to demonstrate performance of the proposed iterative schemes.

1. Introduction

The k-eigenvalue problem for the neutron transport (Boltzmann) equation is given by

$$\mathbf{\Omega}\cdot\nabla\psi + \Sigma_t\psi = \mathscr{S}_s\psi + \frac{1}{k}\mathscr{S}_f\psi, \tag{1}$$

where $\psi = \psi(\mathbf{r}, \mathbf{\Omega}, E)$ is the neutron angular flux, $\mathbf{r}$ is the spatial position, $\mathbf{\Omega}$ is the direction of particle motion, E is the particle energy. The operator on the left-hand side accounts for particle streaming and collisions. The integral operators $\mathscr{S}_s$ and $\mathscr{S}_f$ describe scattering and fission processes. The multiplication factor of a physical system is the largest eigenvalue. The k-eigenvalue problem (1) is used to determine critical parameters of nuclear reactors.

There exist several iterative approaches for solving the multigroup transport equation. One family of methods is based on the synthetic acceleration approach (Adams and Larsen, 2002). An example of such a method is the two-grid acceleration scheme developed to speed up iterations in fixed-source transport problems with upscattering (Adams and Morel, 1993). This method uses a one-group diffusion problem for the iterative error. The one-group diffusion coefficient and cross sections are defined by means of a special spectral shape function that approximates the Fourier harmonic of the solution converging at the slowest rate.

A different group of iterative schemes applies the nonlinear-projective iterative (NPI) methodology and effectively reduces the dimensionality of the transport problem (Anistratov and Gol'din, 1993). The quasidiffusion (QD) method belongs to this group (Gol'din, 1964). The multilevel QD (MLQD) method for solving multigroup problems is defined by the three-level system of equations consisting of (Gol'din, 1982; Anistratov and Gol'din, 1986; Gol'din et al., 1986; Anistratov and Gol'din, 2011)

- the high-order multigroup transport equations for the group angular flux,
- the multigroup low-order QD (LOQD) equations for the moments of the group angular flux, i.e. the group scalar flux and current,
- the effective one-group LOQD problem for the total scalar flux and current.

This hierarchy of equations is closed by means of exact relations defined by linear-fractional factors which are calculated with the solution of the high-order problem. In the MLQD iteration scheme, the eigenvalue is determined as the solution of the problem with the smallest dimensionality, namely, of the effective one-group

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LOQD equations. Its elements can be applied to different NPI methods for solving the transport equation. This methodology was used to develop the multilevel nonlinear diffusion acceleration (NDA) method (Smith and Rhodes, 2002; Anistratov, 2013). It is defined by means of the multigroup and effective one-group low-order NDA equations. Another version of the multilevel NDA method is formulated on a set of coarsening energy grids (Cornejo and Anistratov, 2016). It was shown that introducing additional coarse energy grids can improve acceleration of transport iterations.

One more way to solve the k-eigenvalue transport problem is to treat it as a generalized eigenvalue problem and apply Nonlinear Krylov acceleration and Jacobian-Free Newton-Krylov methods (Gill et al., 2011;Park et al.,2012; Calef et al., 2013; Willert et al., 2014). The iterative methods developed on such basis demonstrated efficiency of this approach. Multigrid in energy preconditioners have also been used for Krylov solvers (Slaybaugh et al., 2013).

In this paper we present a new multilevel QD method in which the group LOQD equations are formulated on a sequence of coarsening grids in energy. The low-order equations on coarse energy grids are used to accelerate the convergence of transport iterations. Different multigrid algorithms are used to solve this set of low-order equations. The eigenvalue is obtained in the space with the lowest dimensionality, namely, as a solution of a one-group eigenvalue LQOD problem. A consistent discretization of the LOQD equations is developed. It is based on a second-order finite volume scheme and uses special compensation (consistency) terms. Thus the proposed multilevel method is a pure acceleration method. The proposed method can be interpreted as a nonlinear multigrid method and described in terms of projection and prolongation operators (Briggs et al., 2000; Trottenberg et al., 2001). The main difference between the MLQD method and multigrid methods is that the equations of the MLQD method are formulated for the solution. Most multigrid algorithms are based on equations for iterative errors.

The remainder of this paper is organized as follows. In Sec. 2 we define the multigroup LOQD equations and describe their spatial discretization. The proposed multilevel QD method on a set of grids in energy is formulated in Sec. 3. In Sec. 4 we present numerical results for typical reactor-physics test problems. We conclude with a discussion in Sec. 5.

2. Discretization of the multigroup low-order quasidiffusion equations

We consider multigroup k-eigenvalue transport problems with isotropic scattering in 2D Cartesian geometry

$$\begin{aligned}\mathbf{\Omega}\cdot\nabla\psi_g(\mathbf{r},\mathbf{\Omega})+\Sigma_{t,g}(\mathbf{r})\psi_g(\mathbf{r},\mathbf{\Omega})&=\frac{1}{4\pi}\sum_{g'=1}^{N_g}\Sigma_{s,g'\to g}(\mathbf{r})\int_{4\pi}\psi_{g'}(\mathbf{r},\mathbf{\Omega})d\mathbf{\Omega}\\&+\frac{\chi_g(\mathbf{r})}{4\pi k}\sum_{g'=1}^{N_g}\nu_{f,g'}(\mathbf{r})\Sigma_{f,g'}(\mathbf{r})\int_{4\pi}\psi_{g'}(\mathbf{r},\mathbf{\Omega})d\mathbf{\Omega},\quad \mathbf{r}\in G,\end{aligned}\tag{2}$$

$$\phi_g(\mathbf{r},\mathbf{\Omega})\big|_{\mathbf{r}\in\partial G_{vac}}=0\quad\text{and}\quad\phi_g(\mathbf{r},\mathbf{\Omega})\big|_{\mathbf{r}\in\partial G_{ref}}=\phi_g(\mathbf{r},\mathbf{\Omega}^*)\big|_{\mathbf{r}\in\partial G_{ref}}\quad\text{for}\quad\mathbf{n}\cdot\mathbf{\Omega}<0,\tag{3a}$$

$$\mathbf{\Omega}^*\cdot\mathbf{n}=-\mathbf{\Omega}\cdot\mathbf{n},\quad\mathbf{n}\cdot\mathbf{\Omega}\times\mathbf{\Omega}^*=0,\tag{3b}$$

$$g=1,\ldots,N_g,$$

where the standard notations are used. Here ∂G is the boundary of the domain G. ∂G_{vac} is the vacuum part of the boundary, ∂G_{ref} is the reflective part of the boundary and $\mathbf{n}$ is the outward normal of the boundary. The multigroup LOQD equations for the group fluxes and currents are derived by taking the zeroth and first moment of the transport equation (2) (Gol'din, 1964; Anistratov and Gol'din, 2011). They are given by

$$\nabla\cdot\mathbf{J}_g+\Sigma_{t,g}\phi_g=\sum_{g'=1}^{N_g}\Sigma_{s,g'\to g}\phi_{g'}+\frac{\chi_g}{k}\sum_{g'=1}^{N_g}\nu_{f,g'}\Sigma_{f,g'}\phi_{g'},\tag{4}$$

$$\nabla\cdot\left(\mathbb{E}_g\phi_g\right)+\Sigma_{t,g}\mathbf{J}_g=0\,,\tag{5}$$

where

$$E_{\alpha\beta,g}=\frac{\int_{4\pi}\Omega_\alpha\Omega_\beta\psi_g d\mathbf{\Omega}}{\int_{4\pi}\psi_g d\mathbf{\Omega}},\quad\alpha,\beta=x,y\tag{6}$$

are the components of the QD (Eddington) tensor $\mathbb{E}_g$ that is defined to close the system of high-order transport and LOQD equations (2), (4) and (5). The boundary conditions for the LOQD equations are the following:

$$\mathbf{n}\cdot\mathbf{J}_g\big|_{\mathbf{r}\in\partial G_{ref}}=0,\quad\mathbf{n}\cdot\mathbf{J}_g\big|_{\mathbf{r}\in\partial G_{vac}}=C_{n,g}\phi_g\big|_{\mathbf{r}\in\partial G_{vac}},\tag{7}$$

where the boundary factor is defined as

$$C_{n,g}=\left.\frac{\int_{\mathbf{\Omega}\cdot\mathbf{n}\geq0}\mathbf{n}\cdot\mathbf{\Omega}\psi_g d\mathbf{\Omega}}{\int_{\mathbf{\Omega}\cdot\mathbf{n}\geq0}\psi_g d\mathbf{\Omega}}\right|_{\mathbf{r}\in\partial G_{vac}}.\tag{8}$$

We consider rectangular spatial grids and discretize the LOQD equations by means of a second-order finite volume method. The balance equation (4) is integrated over each cell (i,j) to obtain

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